# Linear Implicit AC PF Cascading Failure Analysis with Power System Operations and Automation

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***Abstract*— This paper proposes a cascading outages model which captures power system operations, protections, automatic regulations, and remedial action schemes. It is based on linear implicit AC PF, which provides a tradeoff between computationally expensive AC PF and approximate DC PF. Combinations of several types of failures and interactions in blackouts are represented, i.e., cascading line overloads, large frequency deviation, reactive power problems and voltage drop. Statistical comparisons demonstrate that the linear AC PF-based model enhances the accuracy of the power flow solutions as compared to the DC PF with minimum increase of computation time. The case study illustrates that the AC PF-based model provides a more realistic estimate of the consequences of cascading events as compared to the optimistic estimates by DC PF-based models. The computational efficiency and solution accuracy of linear AC-PF model can be exploited in the estimation of the cascade risk for a large number of scenarios.**

***Index Terms*—linear AC power flow, cascading outages simulation, power system operations and automation.**

## I. INTRODUCTION

The reliability of the electric power supply is challenged by the recent deployment of intermittent energy sources in remote areas and by the market liberalization, which resulted in large long-distance power transfer that frequently exceed transmission capacity and increasingly outpace the expansion of transmission network facilities. The increased interconnectivity among neighboring control areas and the integration of volatile renewable energy sources (RES) enhance the risk of cascading failures in power systems [1].

In order to mitigate cascading outages, it is important to develop models and simulations which can provide insight into the complex nature of operation characteristics of electric power systems. Several models build on static security assessment to simulate the magnitude of cascading failures. Depending on the implemented approach, they can be classified as DC PF-based models, such as hidden failure model, ORNL-PSerc-Alaska (OPA) model and AC PF-based models, such as the Manchester model [2].

Due to the large number of possible contingencies, the system operator should be able to quickly estimate the potential occurrence and impact of a cascading event in the transmission network. Model details must be traded off with the simulation time and accuracy, yet, a realistic assessment of cascading failures cannot neglect the effects of real-time operations, protections and automatic regulations which heavily influence and drive the propagation of perturbations and outages. AC PF-based models provide an approximation of the power flow condition which is closer to actual operations. However, this method has high requirement on the computation speed, and may not be feasible for real time applications nor is it appropriate when a multitude of scenarios (contingencies and load profiles) need to be evaluated. The linear nature of DC PF method leads to efficient computation and applicability to large-scale systems. However, due to the complex nature of the power system and cascading failures, the underlying assumptions in DC PF-based cascading failure models may fail. For instance, the voltage profile of buses and reactive power have significant impacts on the system conditions when perturbed by failure events and disregarding their effects may provide too optimistic results and threaten operation security [3]. To mitigate the computational burden of AC PF calculations, a linear implicit AC PF model was proposed for fast power system analysis [4] that strikes a balance between modeling accuracy as well as computational tractability.

In this paper, a cascading outages simulation model which captures power system operations and automations is developed. Our analysis is based on the linear implicit AC power flow model to benefit from modeling accuracy as well as computational speed. Combinations of several types of failures and interactions that typically occur in large blackouts, i.e., cascading line overloads, frequency deviation, reactive power problems and voltage drop, are modeled. The dynamics of operations, protections and automation which guide the system and steer the cascading failure propagation at multiple

time scales are captured, i.e., variation of the load request and ambient condition with time, transient of line temperature, primary and secondary frequency control, and under-frequency load shedding (UFLS). Since our model is based on an underlying AC power flow, we also model voltage instabilities as well as corrective actions such as under-voltage load shedding (UVLD). Due to inclusion of realistic operations, the developed model allows estimating the time that is available for implementing corrective actions before a new topology change or component outage occurs during a cascading event.

The remainder of the paper is structured as follows. Section II introduces the algorithm for cascading failure simulation and details the features that included in the model. In Section III, the performance of the simulation model is benchmarked with case studies on RTS96 and IEEE 118 bus system. Finally, Section IV draws conclusions.

## II. MODEL DESCRIPTION

### A. *Algorithm*

An approximate model which simulates the steady-state operations of the electric network and considers lines overload and tripping, frequency control and protections and voltage protections is developed. The model captures the following real-time operations, protections, automatic regulations, as well as remedial action schemes: (1) yearly load curve for sampling the load demand; (2) critical scenarios which may trigger the cascading event; (3) identification of island operation; (4) primary and secondary frequency control of generators are employed to restore the power generation/consumption balance and eliminate frequency deviation in case of power imbalance in the system or in the generated islands [5]; (5) UFLS is conducted when the frequency exceeds the acceptable threshold [6]; (6) UVLS is performed when the voltage magnitude at a bus is under the limit [7]; (7) it simulates the transient dynamic of line temperature and performs automatic disconnection when the line reaches its failure threshold. Note that in comparison to most conventional cascading failure analyses we additionally model full AC contingencies as well as a wide-range of control and protection mechanisms to counter them.

The consequences of the cascading event are measured as the demand not served (DNS) to the customer, which is a commonly used reliability measure for power system [5].

The flowchart of the simulation model is shown in Fig 1. In the simulation model, the system is initialized with the load condition corresponding to the occurrence time of a cascading event. The initial generation dispatch is obtained from an optimal power flow. Random *k* component failure can be selected as the scenarios to trigger the cascading events. After a topology change, island operation is screened. In the generated island, power imbalance may occur which leads to frequency deviation. Additionally, the inability of the system to meet the demand for reactive power can cause voltage drop. The frequency deviation in the island and the voltage magnitude on each bus are evaluated with linear AC PF method. The primary frequency control will react automatically to reduce the frequency deviation. In case of large frequency deviation and voltage drop that can lead to system collapse, the automated protection of UFLS and UVLS are conducted respectively to bring frequency and voltage magnitude back to acceptable ranges and secure system stability. The load shedding acts on timescales well under one second [8]. The remaining frequency deviation will be compensated by the generators in this island through secondary frequency control. The failure of transmission lines is assumed to be caused by overheating due to sustained excessive power flow. The evolution of the line temperature and its dependence on the carrying electric current are determined by the IEEE Std 738-2012 transient line temperature model [9]. The phenomena of load shedding and line flow redistribution are considered to occur instantaneously when compared to the dynamics of generator ramp up/down and line temperature.

The model is characterized by a time-base structure, where 3 types of events are defined. The simulation advances in time based on the parallel evolution of these three events:

1) Time interval changes: at each new hour, a load demand is sampled from the load curve. The simulation time moves forward to next hour. The chosen time interval can be refined if needed.

2) Continuous time evolution: as a result of secondary frequency control, generators ramp up/down to restore the power balance and bring the frequency back to the nominal value, the simulation time moves to the time point when frequency deviation is eliminated.

3) Discrete events: the line thermal dynamic model predicts the evolution of the line temperatures and estimates the failure time when to disconnect a line, due to overheating protection. The line with shortest failure time will be tripped and the simulation time moves to the time point when this line is tripped.

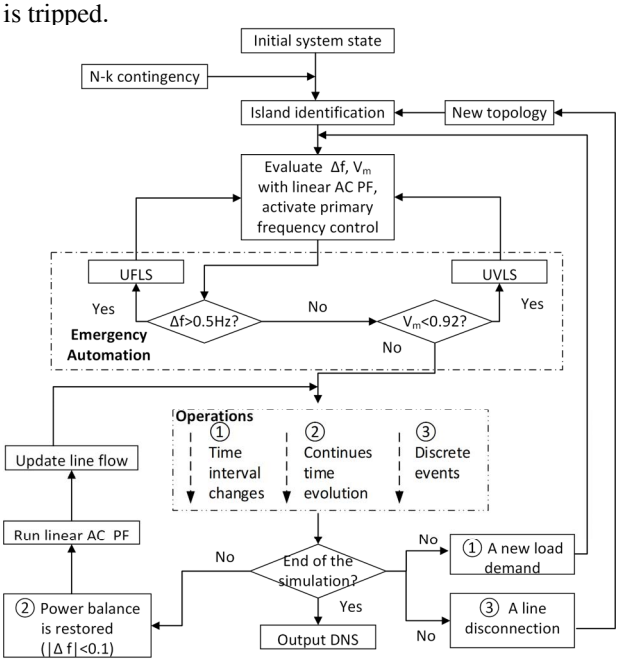



Figure 1. Flowchart for the model's algorithm

The next event determines the system change to occur. If it is a new hour, the load demand is updated; if it is a topology change, island formation is assessed. In both cases, the changes to the frequency and voltage conditions are checked, the activation of protections is assessed, and power balance is established in each island via frequency control and load shedding. After power imbalance is compensated, linear AC PF algorithm is applied for evaluating the new line flows, and the evolution of line temperatures and the time needed for the lines to reach failure temperature is updated via the transient line temperature model. Depending the time it takes for each event to happen, the next event would be a new load demand, the disconnection of the line with minimum failure time or a new power balance condition. The cascading events terminate when there is no event is scheduled to happen. At the end of the simulation, DNS caused in the cascading outages is output.

### *B. Formulation of the Linear AC Power Flow*

The linear implicit AC PF method proposed in [4] is applied in the cascade model for fast power flow computation. The method consists of two parts: the first part implements the power grid equations without bus models, i.e., all power injections and voltages that are considered as variables compatible with the network physics independent of specific bus models that constrains these variables such as PV, PQ, or slack buses. The power flow manifold $M$: $F(x) = 0$ represents the set of nonlinear and complex-valued power flow equations that descend directly from Kirchhoff's and Ohm's laws, and the variables $x$ ($\upsilon$, $\theta$, $p$, $q$) include the grid state of voltage magnitude $\upsilon$, voltage angle $\theta$, active power injection $p$, reactive power injection $q$. A linear local approximant for the manifold $M$ of all feasible power flows is obtained by deriving the tangent plane at a feasible point $x^* \in M$ compatible with the network physics. The feasible point $x^*$ can be chosen as the operating point before the initial contingency, which gives the best possible linear approximant as the linear manifold tangent to $M$ in $x^*$ by

$$A_{x^*}(x - x^*) = \mathbf{0}_{2n} . \tag{1}$$

The second part of the method implements the standard models for grid buses. For each bus $h$, there are equality constraints depending on the type of bus

$$\text{PQ buses: } g_h(\mathrm{x}_h) = \begin{bmatrix} p_h - P_h \\ q_h - Q_h \end{bmatrix} = \mathbf{0}_2 \tag{2}$$

$$\text{PV buses: } g_h(\mathrm{x}_h) = \begin{bmatrix} p_h - P_h \\ \upsilon_h - V_h \end{bmatrix} = \mathbf{0}_2 \tag{3}$$

$$\text{Slack bus: } g_h(\mathrm{x}_h) = \begin{bmatrix} \upsilon_h - V_h \\ \theta_h \end{bmatrix} = \mathbf{0}_2 . \tag{4}$$

Eq. (1)-(4) provide a sparse and implicit linear approximant for the *initial* model before contingencies take place. Yet, we work with this model also during the development of the cascade and do not update the operating point $x^* \in M$ and the resulting linearization to avoid costly AC power flow computations. Nevertheless, we observe that the linear model (1)-(4) performs excellent compared to expensive repeated nonlinear AC power flow evaluations.

### *C. Frequency Control*

The system frequency can deviate from the nominal value during a disturbance, e.g., when an area with large generation capacity is isolated. In our model, the frequency deviation is added as an additional variable in system state $x$, and it is computed by modifying the grid buses model in the linear AC PF formulation. For all the buses with generators (PV and slack bus), the equality constraints are modified according to the action of speed droop control as

$$\text{PV buses: } g_h(\mathrm{x}_h) = \begin{bmatrix} p_h + \Delta f / R_{g,h} - P_h \\ \upsilon_h - V_h \end{bmatrix} = \mathbf{0}_2 \tag{5}$$

$$\text{Slack bus: } g_h(\mathrm{x}_h) = \begin{bmatrix} p_h + \Delta f / R_{g,h} - P_h \\ \upsilon_h - V_h \\ \theta_h \end{bmatrix} = \mathbf{0}_3 \tag{6}$$

where $\Delta f$ is the frequency deviation at the steady state in Hz and $R_{g,h}$ is the frequency characteristics of the generator connected at bus $h$ in Hz/MW, respectively. The power generated at bus $h$ consists of the nominal power generation $p_h$ and the part contributed by the primary frequency control, i.e. $\Delta f / R_h$. UFLS is implemented when the frequency deviation exceeds a safety threshold. According to [10], 40% of load is to be shed if frequency reaches 49.5Hz, 70% at 49Hz and 100% at 48.5Hz.

When system frequency is stabilized, secondary frequency control is activated to restore frequency to the nominal value. Here we restrict ourselves to a single-area system, where the objective of secondary control is power balancing within the area. The contribution of each generator to secondary frequency control is computed from the nodal balance at bus $h$

$$0 = P_{g,h}^{\text{nom}} + P_{g,h}^{\text{primary}} + P_{g,h}^{\text{secondary}} - P_{d,h} - \sum_{k \in n_h} P_{l,hk} \tag{7}$$

where $P_{g,h}^{\text{nom}}$ is the nominal power generation, $P_{g,h}^{\text{primary}} = -\Delta f / R_h$ and $P_{g,h}^{\text{secondary}} = u_{g,h}$ are the power delivery for primary droop control and secondary control. $P_{d,h}$ is the power demand at bus $h$, $n_h$ is the set of the buses that are connected to bus $h$ and $P_{l,hk}$ is the flow on line {h,k}, respectively. By summing up eq.(7) for all the buses we obtain

$$\Delta f = \left( \sum_{h=1}^{N_{\text{bus}}} P_{g,h}^{\text{nom}} + \sum_{h=1}^{N_{\text{bus}}} u_{g,h} - \sum_{h=1}^{N_{\text{bus}}} P_{d,h} - \sum_{h=1}^{N_{\text{bus}}} \sum_{k \in n_h} P_{loss,hk} \right) \cdot \sum_{h=1}^{N_{\text{bus}}} R_h \tag{8}$$

where the left-hand side denotes the *measurable* system frequency deviation $\Delta f$, the term $P_{loss,hk}$ depicts the power loss on the transmission line {h,k}, and the term in brackets on the right-hand side denotes the total power imbalance. Observe that $\Delta f$ is zero if and only if the power injections and losses are balanced by means of the secondary control input $u_{g.h}$. The secondary control action $u_{g.h}$ is implemented via integral control of the frequency error $\Delta f$ with integral gain equal to the generation participation factor $\beta_{g,h}$ for each generator, which represents the ratio of generator $h$'s capacity to the total capacity, i.e., $\beta_{g,h} = P_{g,h}^{\text{capacity}} / \sum_{h=1}^{N_{\text{bus}}} P_{g,h}^{\text{capacity}}$. Thus, the steady-state of the secondary integral control input achieving $\Delta f = 0$ is given by

$$u_{g,h} = \beta_{g,h} \cdot \left( \sum_{h=1}^{N_{bus}} P_{d,h} + \sum_{h=1}^{N_{bus}} \sum_{k \in n_h} P_{loss,hk} - \sum_{h=1}^{N_{bus}} P_{g,h}^{nom} \right) \quad (9)$$

### D. Under Voltage Load Shedding

A system may enter a state of voltage instability when it is subject to a disturbance, e.g. increase in load demand, sequential line tripping or other change in system condition that can lead to progressive and uncontrollable drop in voltage. The steady-state criteria is that the post-contingency voltage is above 92% of nominal or 0.92 p.u [7]. One of the most economical remedial actions to voltage drop is UVLS [11]. In order to implement UVLS, its location and amount have to be identified. In the model, UVLS is applied to the load directly connected to the bus with under voltage [11]; at each iteration, 25% of the initial load is shed and the voltage magnitude is updated by linear implicit AC PF. The iteration continues until the voltage magnitude is restored back to 0.92 p.u.. If UVLS cannot restore the voltage back to 0.92 p.u., the entire load connected to the bus is lost.

## III. RESULTS

To test the performance of the simulation model, computational experiments are performed on the RTS 24-bus system [12] and IEEE 118-bus system [13]. First, the approximation accuracy and the computational requirements are compared against full AC PF and DC PF methods, which expect to be either computationally expensive or perform relatively inaccurately. Next, the difference of the cascading consequences evaluated in the simulations based on the full AC, linear AC PF and the DC PF methods is explored.

### A. Approximation Error and Simulation Time

In order to simulate different operating conditions, statistical studies are conducted by initiating cascading outages in the proposed linear AC PF based model with random multiple line contingencies, i.e., 1000 simulations for 24 bus system. At every stage of the cascading simulations, the linear implicit AC PF method is applied for updating the line flows. For comparison purposes, the full AC and DC PF power flow solutions are computed as well, and the solutions of the linear AC and DC PF methods are compared to the solutions of the standard AC method. The relative approximation error of the power flow on line $l$ is calculated as

$$\varepsilon_l^{\text{linear AC/DC}} = \frac{\left| P_l^{\text{linear AC/DC}} - P_l^{\text{AC}} \right|}{P_l^{\text{capacity}}} \quad (13)$$

where $P_l^{\text{linear AC/DC}}$ and $P_l^{\text{AC}}$ are the power flow calculated with linear AC/DC method and full AC method, respectively, $P_{f,l}^{\text{capacity}}$ is the transmission capacity of line $l$. Fig 2 illustrates the average approximation error obtained from 1000 simulations of the power flow on each line, i.e., $\varepsilon_l^{\text{DC}}$ and $\varepsilon_l^{\text{linear AC}}$. It can be observed that the results are significantly more accurate when the power flow is evaluated with the linear AC PF method instead of the DC PF method despite using only a single linearization point throughout the entire cascade.

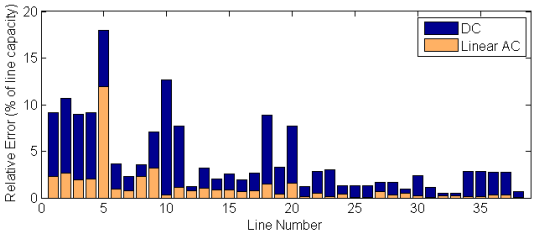



Figure 2. Average approximation error of linear AC and DC method in the apparent and real power flow, respectively, for each line in the 24 bus system. Every PF calculation performed contributed to the average error.

The computation time for repeatedly solving the PF equations throughout the cascade is analyzed for each method. Fig 3 shows the percentage of computation time reduction from full AC to linear AC method and from full AC to DC method. The DC method reduces 90% of the computation time compared to full AC method. For linear AC method, it can reduce 80-90% of computation time with respect to the full AC method. The results thus far in Figures 2 and 3 demonstrate that the linear AC method allows for significantly more accurate results at the expense of a minimal increase in the computational effort as compared to DC PF method.

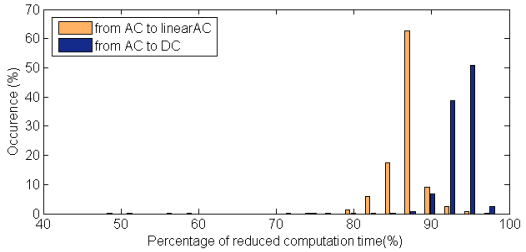
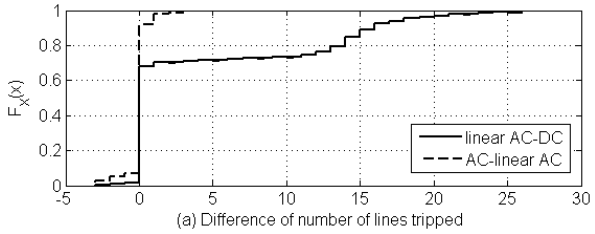



Figure 3. Distribution of the reduced computation time in the 24 bus system

### B. Linear Implicit AC vs. DC Cascading Failures

Compared to a DC PF based simulation model, the proposed linear AC PF simulation model includes the post-fault reactive power flows and voltage at buses. In order to study how the different model accuracy can affect the system states change, the line tripping sequence, and, ultimately, the propagation process, the cascading events initiated by the same contingencies are simulated with full AC, linear AC and DC PF models. 2000 simulations with initial conditions of random $N$-3 line contingencies and varying load conditions (increase the load level by 60% and 80%) are conducted in the IEEE 118 bus system. The consequences of the cascading outages are quantified by the number of lines tripped, the number of cascade stages, and the DNS. Fig. 4 shows the cumulative distribution functions ($F_X(x)$=Prob{$x \leq X$}) of the difference of the consequences estimated by the three models.



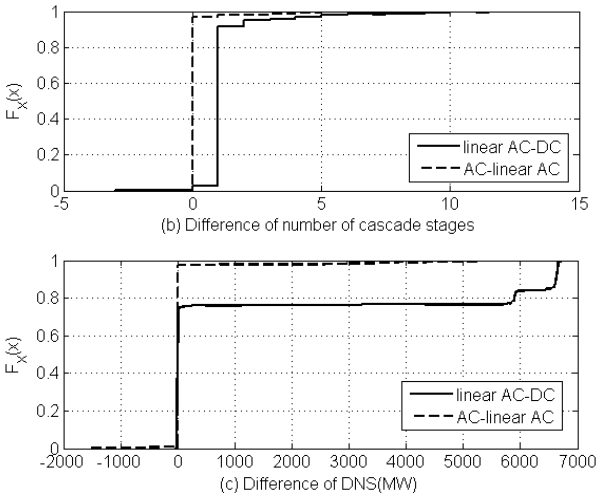



Figure 4. Cumulative distributions of the difference of the consequences estimated in full AC, linear AC and DC PF-based cascading outages models. (a) difference of number of lines tripped (b) difference of number of cascade stages(c) difference of DNS.

Fig.4 (a) shows that linear AC PF-based cascading failures result in more line trips and more cascade stages in 30% of the cascading events as compared to DF PF-based cascades. These results are in line with the general observation that the DC PF is *optimistic* compared to the AC PF, in the sense that lines are less stressed and thus DC PF models underestimate outages. Fig. 4(b) shows that generally DC PF-based cascading outages stop at a certain simulation stage, while linear AC PF-based cascading failures continue to propagate and may finally lead to system blackout. This is due to the voltage dynamics and reactive power flows that become very prominent when lines are severely overloaded, and the resulting voltage differences partially compensate for an active power overload. Additionally, cascades can be extended by reactive power control actions such UVLS mitigating voltage violations. Fig.4 (c) shows that DNS caused by linear AC PF-based cascades exceeds the DNS caused by DC-based cascades by 5800-6600MW in 20% of the cascading events. This difference is close to the initial power demand in the system, i.e. 6721MW. Therefore, the DC PF method with lower approximation accuracy fails to identify the overloaded lines and the potential cascading outages, and, as a result, it may underestimate the consequence of cascading events. Conversely, the comparison of the results of linear AC and full AC PF-based models in Fig. 4(a)-(c) shows that linear AC PF-based model provide an extremely good estimate of the consequences of cascading events.

In summary, we find that the linear AC PF model is significantly more accurate than the DC PF at a marginal increase in computational cost. The linear AC model provides significantly more accurate line flows and takes voltage magnitudes and reactive power flows and the accompanying control actions into account. As a result, the cascading paths governed by these two power flow models may be severely different, where the DC PF results are biased towards overly optimistic outcomes possibly giving the operator a skewed assessment of the cascade. Hence, we propose to use the more accurate linear AC PF model and take additional effects, voltage violations, and corrective actions into account, as these extensions require only a marginally increase computational effort.

## IV. CONCLUSIONS

In this paper, a novel cascading outages simulation model is proposed to study the evolution and to estimate the consequence of cascading outages. It is based on linear implicit AC PF, which provides a tradeoff between computationally expensive AC PF and approximate DC PF. The developed model includes the mechanisms which guide the propagations of cascading outages, i.e. the reactive power problems and voltage drop, which cannot be modeled by DC PF-based cascade model. Statistical comparisons demonstrate that linear AC PF enhances the accuracy of the power flow solutions as compared to the DC PF with minimum increase of computation time. The case study illustrates that the consequence of a cascading event may be underestimated using the DC PF-based model. In the future work, the application of the proposed cascading outages model will be explored. Thanks to its computational efficiency, it is promising tool for online risk assessment of different economic power dispatches.

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