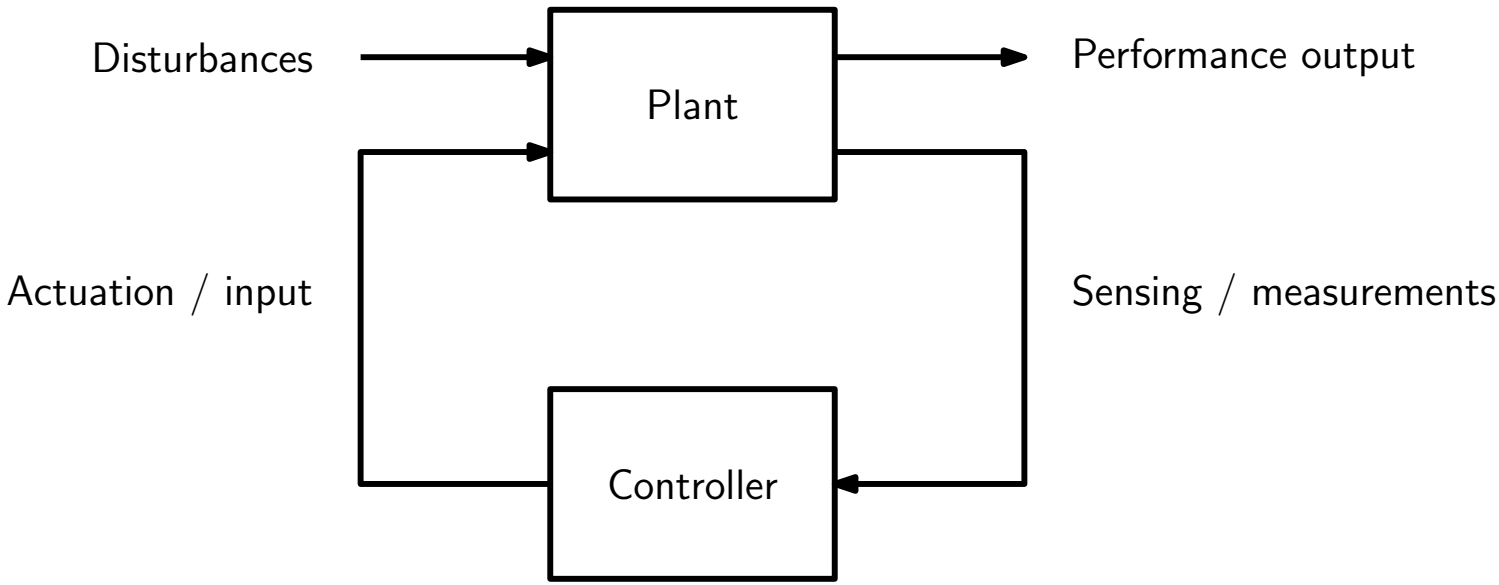
Computational Control

Intro

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Automatic Control Laboratory (IfA)
ETH Zurich


Disturbances
Plant
Performance output
Actuation / input
Sensing / measurements
Controller

A room full of control experts

- Look around you, form little groups (3-4 people).
- You are a team of **experts in design of control systems**.
- Nominate **one person** that is responsible to
  1. keeping time
  2. making sure everybody has the chance to contribute
  3. representing the team
- Use **5 minutes** to check what competences you have in your team.
  - What **methods** to you know to design a control system?
    - **math formalism:** differential equations, state space, transfer functions, games, ...
    - **modeling:** first-principle models, identification, data fitting, black box, ...
    - **control architectures:** feedback, feedforward, multi-agent, distributed, ...
    - **analysis tools:** stability analisys, linear / nonlinear, spectral, stochastic, ...
    - **design methods:** pole placement, frequency, optimal control, tuning methods, ...

    No need to write down anything.







ECB raises interest rates across eurozone by record margin

European Central Bank sets aside recession fears to increase rates by 0.75 of a percentage point to 1.25%



The ECB president, Christine Lagarde, in Frankfurt on Thursday. The bank had to respond forcefully to criticism of falling behind the curve, said one investment strategist of the rates decision. Photograph: Kai Pfaffenbach/Reuters

The European Central Bank has raised interest rates across the eurozone by a record margin to combat soaring inflation that has reached double figures in some of the currency bloc's 19 member countries.

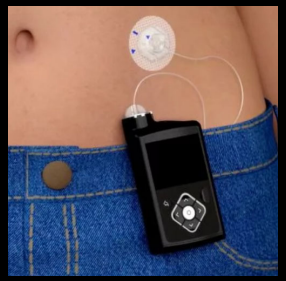




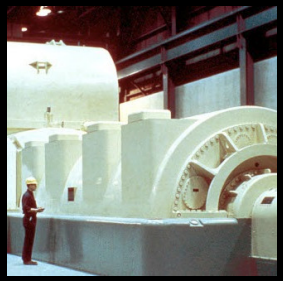








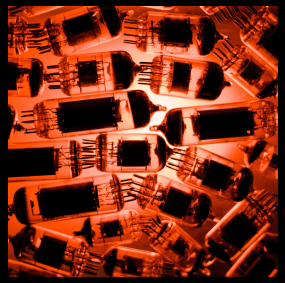




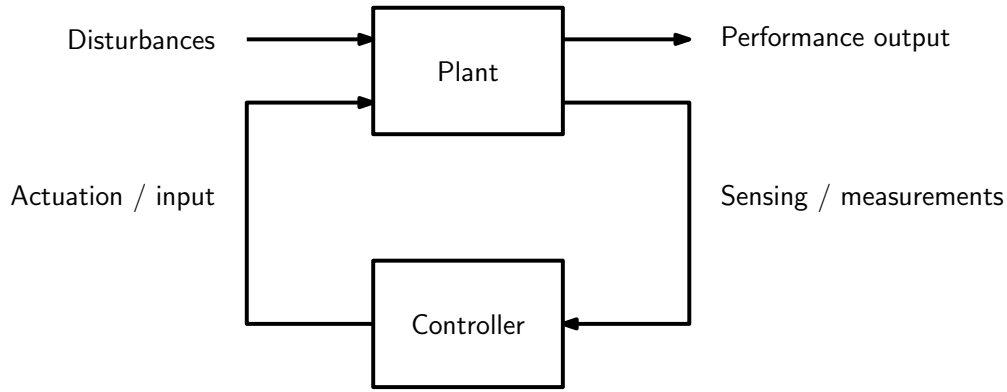








- We will go through these 8 examples one by one.
- For each example I will describe the **plant** and present a **control challenge**.
- Complete the block diagram
  - what is the plant
  - what is the input
  - what is measured
  - **what is the controller**



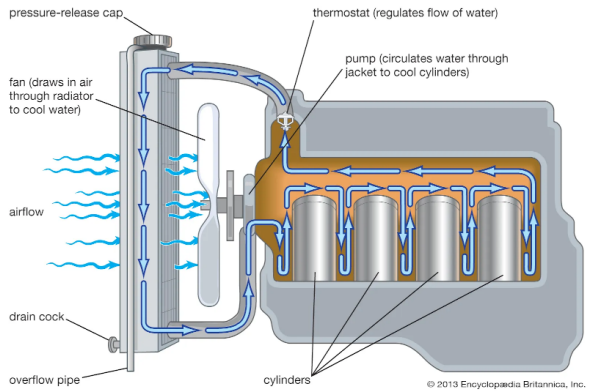
- Discuss how you would attack the challenge, given your team's competences.
  - Do you think you have the necessary competences?
  - Would you take up the challenge?
  - If **yes**, what methods would you use?
  - If **no**, what's blocking you?

Quick! 3 minutes per case.

Case 1







- Your job is to design the thermostatic valve that regulates the flow of coolant through a car engine and a car radiator.
- **Goal:** design the valve to ensure that the temperature of the engine lies in the desired range.

Case 2





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The ECB president, Christine Lagarde, in Frankfurt on Thursday. The bank had to respond forcefully to criticism of falling behind the curve, said one investment strategist of the rates decision. Photograph: Kai Pfaffenbach/Reuters

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$$i = (r^* + \pi) + a\,(\pi - \pi^*)$$

Fisher equation (defines long-run equilibrium)

Deviation of inflation from price stability

- As of today, central banks adjust their interest rate according to Taylor's formula
  - i: interest rate
  - r^*: natural interest rate
  - π^*: desired inflation rate
  - π: inflation rate
- The underlying assumption is that low interest rates promote borrowing, which causes inflation (and viceversa)
- **Goal:** study the stability of this rule and propose a better one.

Case 3





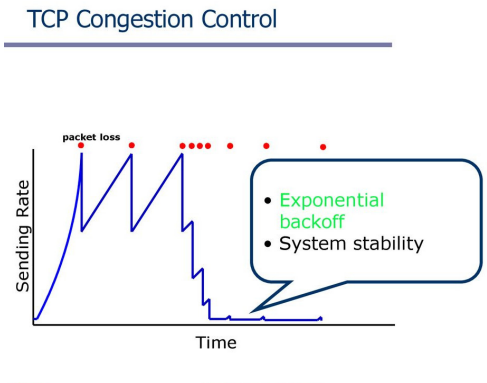


- Autonomous trucks are expected to form a platoon.
- The truck speed should be regulated based on the measured distance from the truck in front.
- **Goal:** design a cruise controller to ensure safety and minimize fuel use.

Case 4





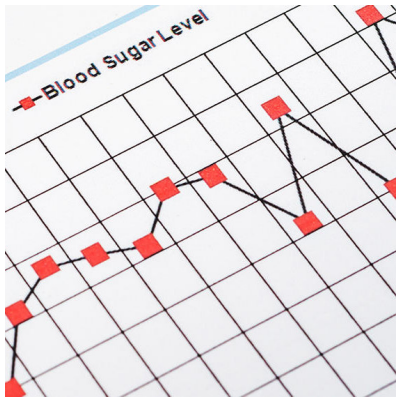


- Each computer in a data network needs to decide its transfer rate.
- Concurrent transmissions by multiple computers cause congestion in the network, which needs to be prevented.
- In today's protocols, individual computer slow down when they detect congestion (e.g., lost packets) and speed up otherwise.
- **Goal:** improve these protocols and guarantee efficient use of the data network.

Case 5



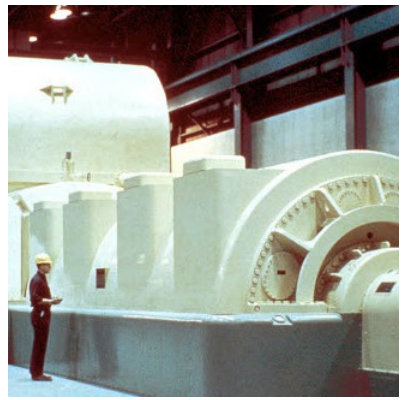




- A healthy pancreas produces insulin when a high level of sugar in the blood is detected, to maintain it in an optimal range.
- In people affected by diabetes, the pancreas does not perform this function properly. It is possible to assist them with an artificial pancreas that releases insulin when needed.
- The artificial pancreas can measure the sugar in the patient blood and the patient can signal when they are having a meal. The level of sugar needs to stay in a prescribed range, and absolutely above a critical lower limit.
- **Goal:** design the artificial pancreas logic.

Case 6





change in grid frequency

mechanical power (steam)

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_a = P_m - P_e$$

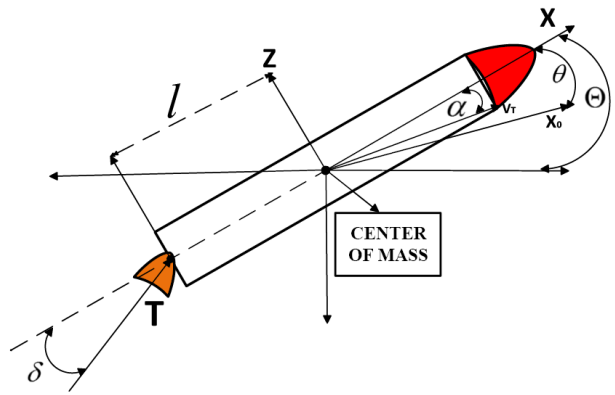
electrical power (to the grid)

- In a power grid it is fundamental to balance generated power and demand
- When there is a mismatch, generators automatically release or absorb kinetic energy by slowing down or speeding up
- Their speed is reflected in the grid frequency, which deviates slightly from 50 Hz
- **Goal:** design a system to adjust the generated power P_m and balance supply and demand

Case 7



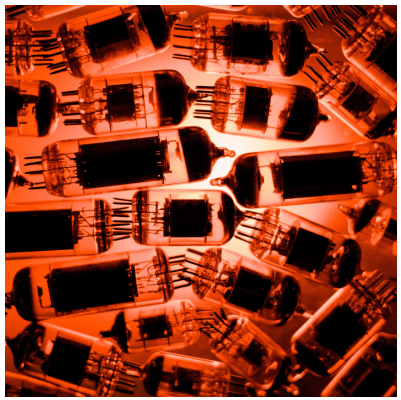


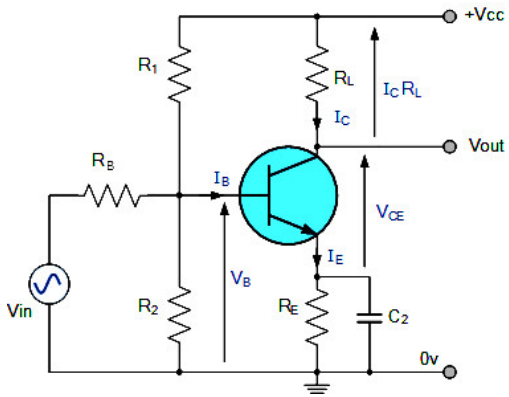


- A rocket launch comes with a predetermined trajectory that needs to be followed.
- The thrust of the rocket can be adjusted to make corrections.
- We want to bring the rocket back to Earth and land it.
- It is imperative to maintain orientation within strict limits and to regulate its speed to exactly zero at the landing point.
- **Goal:** design a rocket landing controller.

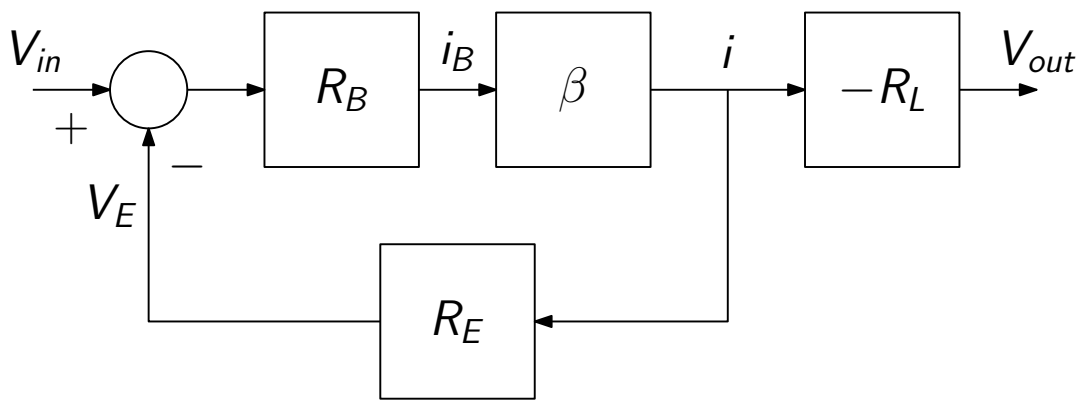
Case 8







- An audio amplifier is made of discrete elements and includes a vacuum tube (power amplifier).
- The amplification β of the tube is very high but uncertain.
- **Goal:** design the circuit to guarantee lower but precise amplification over a large frequency band.



Advanced control design for cyber-physical systems

Cyber-physical systems

Systems in which the controller is an **advanced embedded computer** that can sense and actuate a **physical plant**.





In this course we will

- Focus on designing controllers for complex physical systems
- Look at control problems that PID controllers can't solve ... 5%?
- Exploit the power of real-time computation and data

Automatic Control Lab http://control.ee.ethz.ch



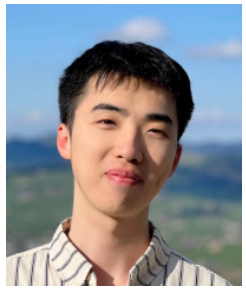
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Mohamed Abdalmoaty



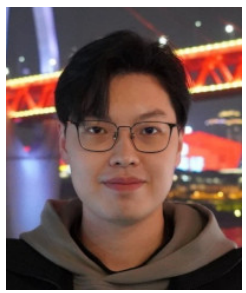
Feiran Zhao (head TA)



Jonas Ohnemus



Mattia Bianchi



Zhisen Jiang



Course logistics

- **Lectures**
  - Every Monday 10:15 - 12:00 in HG D 1.1
- **Tutorials**
  - Every Wednesday 10:15 - 12:00 in HG E 1.2
- **Office hours**
  - Via the Moodle forum
  - For any communication that cannot happen on the forum, via email
- **Course website** on Moodle
  - **Lecture slides** will be there before each lecture
  - **Exercises from the tutorials** + solutions + forum discussion
  - **Announcements** (you are responsible to check)
- **Video recording and streaming**
  - **Lectures only**
  - Restricted access on https://video.ethz.ch, see Moodle for password.

Grading and exam

- **6 credits**
- **Performance assessment**
  - 30% of the final grade is based on an invidividual **final project**
    - Slides, code, mini oral presentation (t.b.a.)
  - 70% based on a **written final exam**
    - No aids, in August (location and exact date t.b.a.)
- **Prerequisites**
  - Control systems fundamentals
  - Linear algebra
- **Useful / will learn as needed**
  - Convex optimization
  - Linear systems theory (reachability, controllability, ...)
  - Lyapunov stability theory

Project

- Students will be provided with a **real-world control problem**.
  - Vertical rocket landing
- The problem description will be provided in the form of natural language description of the desired **specifications**.
- A **numerical simulator** and basic **benchmark control solution** will be provided.

You as a Control Expert

Imagine that you have to **convince** a stakeholder to replace the current controller with an advanced controller that you **recommend**.

In the **project report** (python notebook + slides), you will have to

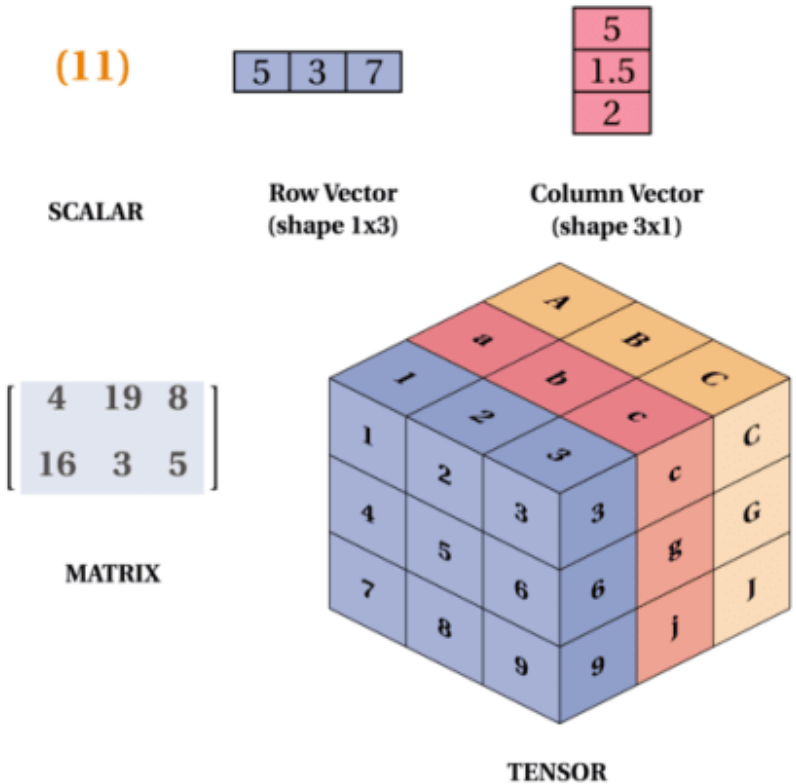
1. Illustrate why the current control strategy is not appropriate.
2. Propose an advanced control strategy among those seen in class.
3. Provide evidence of its effectiveness and appropriateness.

More info soon!

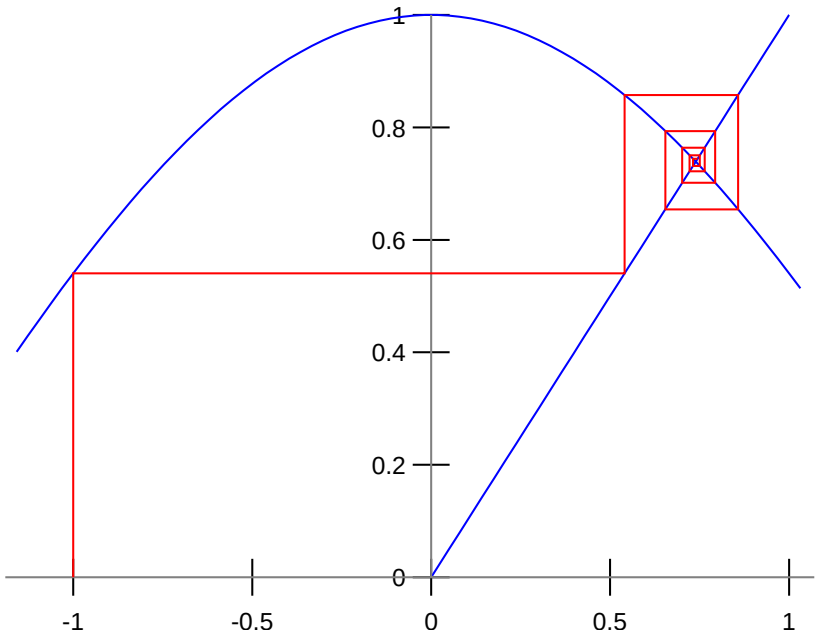
Basic concepts in convex optimization

What are computers good at?

Linear algebra



Fixed point iterations



Mathematical optimization

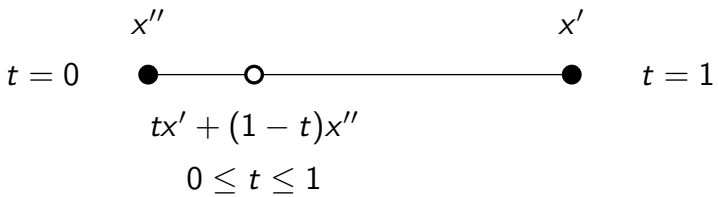
Core decision problem in most computational control schemes

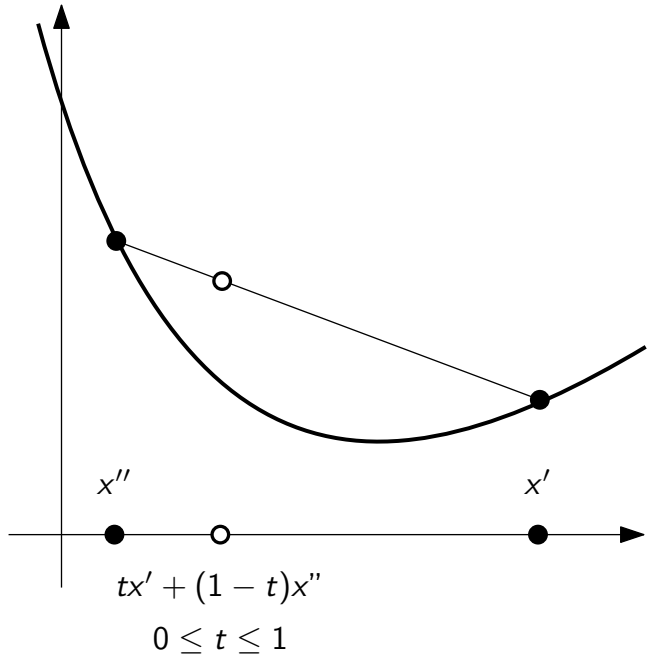
$$\begin{aligned} \text{minimize}_{\ x \in \mathcal{X}} \quad & f(x) \\ \text{subject to} \quad & g(x) \leq \mathbf{0} \\ & h(x) = \mathbf{0}. \end{aligned}$$

- $x \in \mathcal{X} \subseteq \mathbb{R}^n$ is the **decision variable**
- $f : \mathbb{R}^n \to \mathbb{R}$ is a **cost function**
- $g : \mathbb{R}^n \to \mathbb{R}^m$ describes **inequality constraints**
- $h : \mathbb{R}^n \to \mathbb{R}^p$ describes **equality constraints**

When are computers good at solving these in real-time?

Convexity



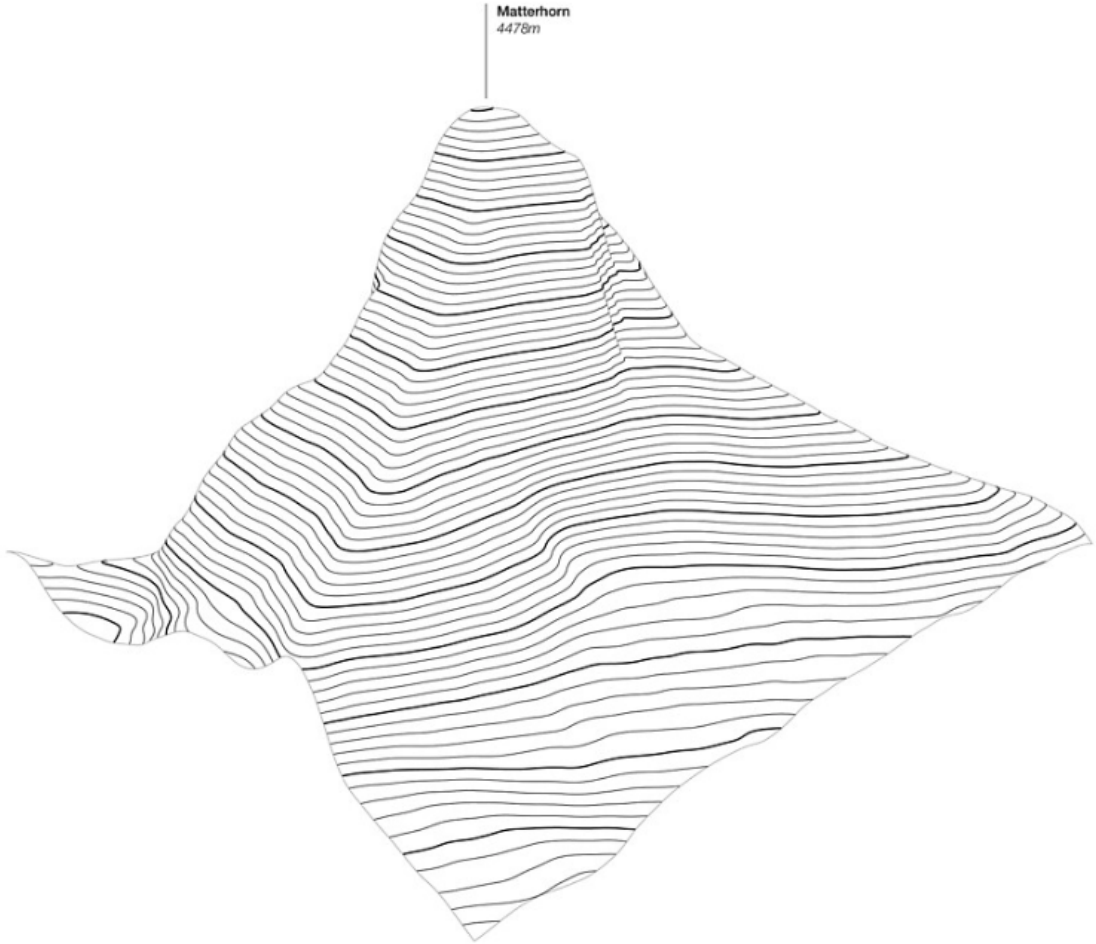


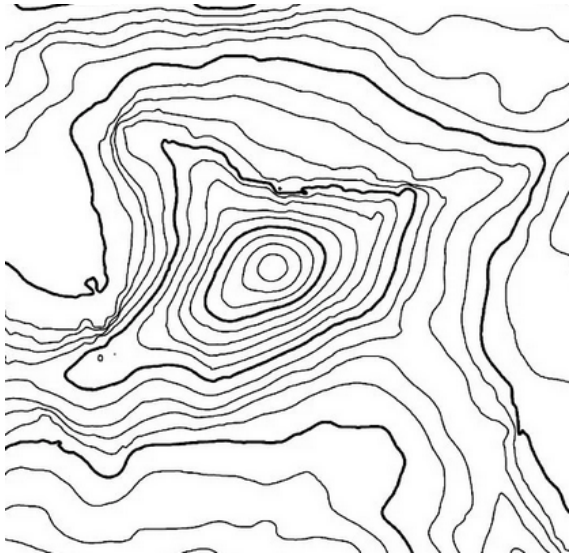
Convex sets

$$x' \in \mathcal{X} \quad \& \quad x'' \in \mathcal{X} \quad \Rightarrow tx' + (1-t)x'' \in \mathcal{X} \qquad \forall\, 0 \leq t \leq 1$$

Convex functions

$$\phi(tx' + (1-t)x'') \leq t\phi(x') + (1-t)\phi(x'') \qquad \forall\, 0 \leq t \leq 1$$





Convex sets and convex functions

$$g(x) \text{ convex} \quad \overset{?}{\Leftrightarrow} \quad \{g(x) \leq c\} \text{ convex}$$

$$g(x) \text{ convex} \qquad \overset{?}{\Rightarrow} \qquad \{g(x) \leq c\} \text{ convex}$$

We need to prove that if

$$x' \in \{g(x) \leq c\} \quad \text{and} \quad x'' \in \{g(x) \leq c\}$$

then for all $0 \leq t \leq 1$, the point $tx' + (1-t)x'' \in \{g(x) \leq c\}$.
That is, we have to prove that $g(tx' + (1-t)x'') \leq 0$.
Because of convexity of $g(\cdot)$, we have that

$$g(tx' + (1-t)x'') \leq tg(x') + (1-t)g(x'') \leq tc + (1-t)c = c.$$

$$g(x) \text{ convex} \qquad \overset{?}{\Leftarrow} \qquad \{g(x) \leq c\} \text{ convex}$$

It's not true, as shown by any of these counterexamples.

$$g(x) = -e^{-x} \qquad g(x) = \sin(x) + x \qquad \ldots$$

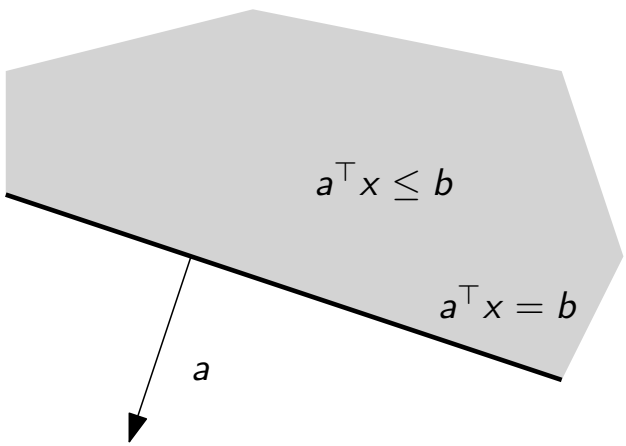
Common convex sets

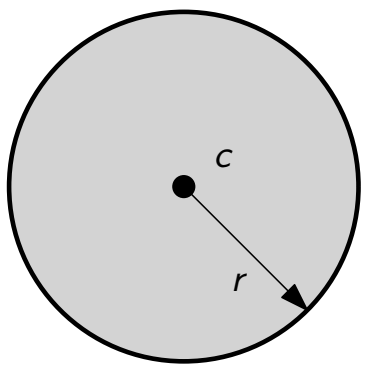
Halfspaces

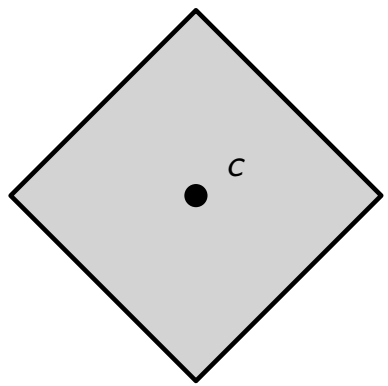
$$\{x \mid a^\top x \leq b\}$$

Balls

$$\{x \mid \|x - c\| < r\}$$







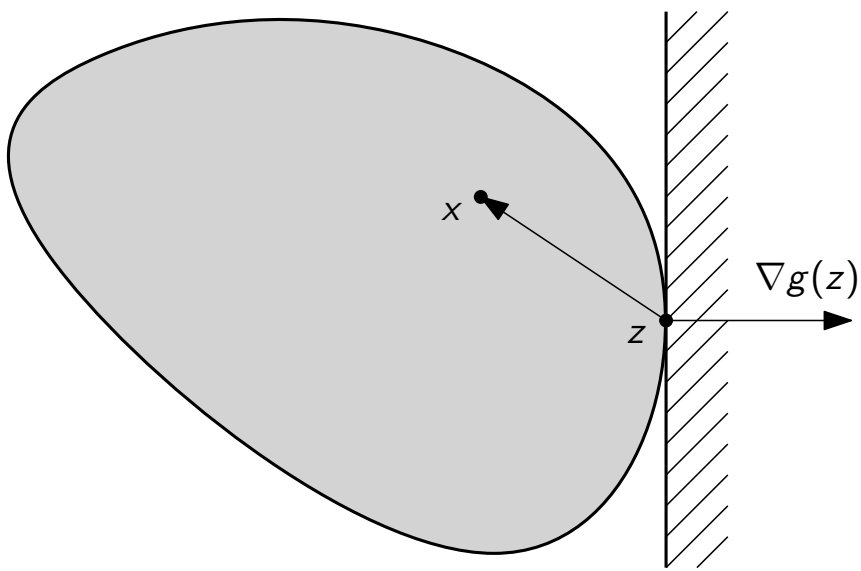
Supporting hyperplanes

Theorem

If a set is convex, then there exists a supporting hyperplane at every point z on its boundary.

If the set is defined as $\{x \mid g(x) \leq 0\}$, then the supporting hyperplane is

$$x \mid \nabla g(z)^\top (x - z) \leq 0 \qquad \text{that is} \qquad x \mid \underbrace{\nabla g(z)^\top}_{a^\top} x \leq \underbrace{\nabla g(z)^\top z}_{b}$$



Unions and intersection of set

- Is the **union** of convex sets a convex set?
- Is the **intersection** of convex sets a convex set?

Try to prove it

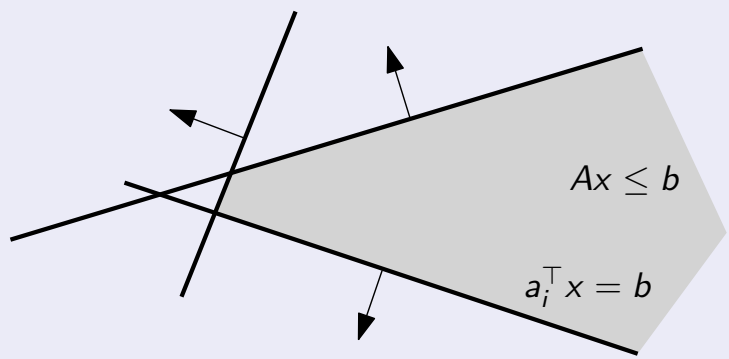
Given a statement, prove that it's true (deductive proof) or false (counterexample).

Polytopes

Intersection of hyperplanes

$$\{x \mid a_i^\top x \leq b \; \forall i = 1, \ldots, r\}$$

$$\{x \mid Ax \leq b\}$$



Common convex functions

- **Affine functions**

$$f(x) = a^\top x + b$$

- **Norms**

$$f(x) = \|x - c\|$$

- **Quadratic functions**

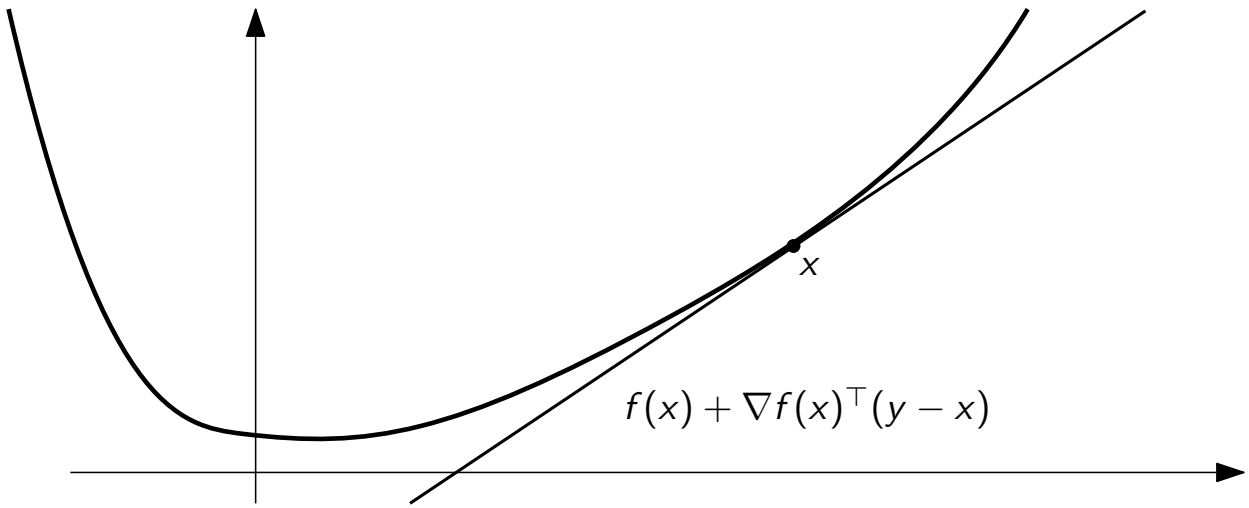
$$f(x) = x^\top A x + b^\top x \qquad A \succeq 0 \text{ (positive semi-definite)}$$

First and second derivatives

First-order condition

If f is **differentiable** on a convex domain, then

$$f \text{ convex} \quad \Leftrightarrow \quad f(y) \geq f(x) + \nabla f(x)^\top (y - x) \quad \forall x, y$$



Second-order condition

If f is **twice differentiable** on a convex domain, then

$$f \text{ convex} \quad \Leftrightarrow \quad \nabla^2 f(x) \succeq 0 \quad \forall x$$

Convex optimization problems

$$\begin{aligned} \text{minimize}_{\ x \in \mathcal{X}} \quad & f(x) \\ \text{subject to} \quad & g(x) \leq \mathbf{0} \\ & h(x) = \mathbf{0}. \end{aligned}$$

- $\mathcal{X}$ convex set
- f convex function
- g convex function
- h affine function

Convex feasible set: $\mathcal{X} \cap \{x \mid g(x) \leq 0\} \cap \{x \mid h(x) = 0\}$

Convex optimization problems are much simpler to tackle computationally compared to general optimization problems.

Why?

Local optimality conditions (1)

x^* is a local optimum if there exists R such that

$$f(z) \geq f(x^*) \quad \text{for all feasible } z \text{ with } \|z - x^*\| \leq R$$

Proposition

Any local optimum of a **convex** problem is a global optimum.

Proof:

Suppose x^* is locally optimal, but there exists y such that $f(y) < f(x^*)$.
Consider $z = ty + (1-t)x^*$ with $t = \frac{R}{2\|y-x^*\|}$. z is feasible, thanks to convexity.
z is inside the ball of radius R, because $\|z - x^*\| = t\|y - x^*\| = R/2$.
Because of convexity,

$$f(z) \leq tf(y) + (1-t)f(x^*) < tf(x^*) + (1-t)f(x^*) = f(x^*),$$

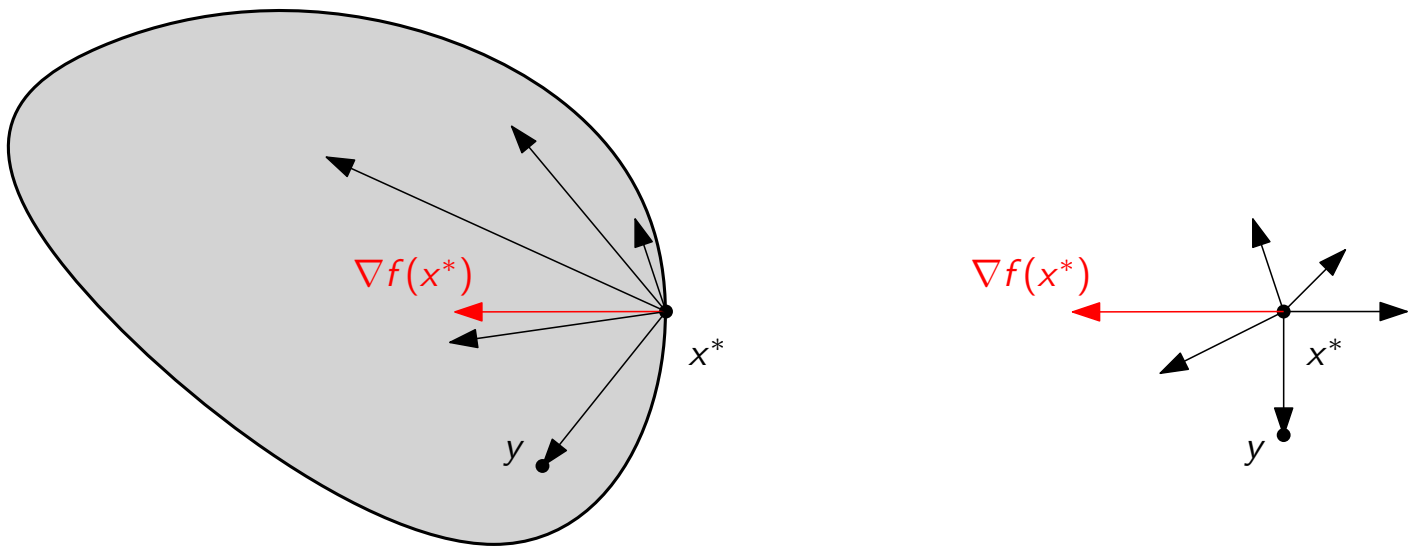
which contadicts the local optimality assumption.

Local optimality conditions (2)

Proposition

x^* is a local optimum for a **differentiable** problem if and only if

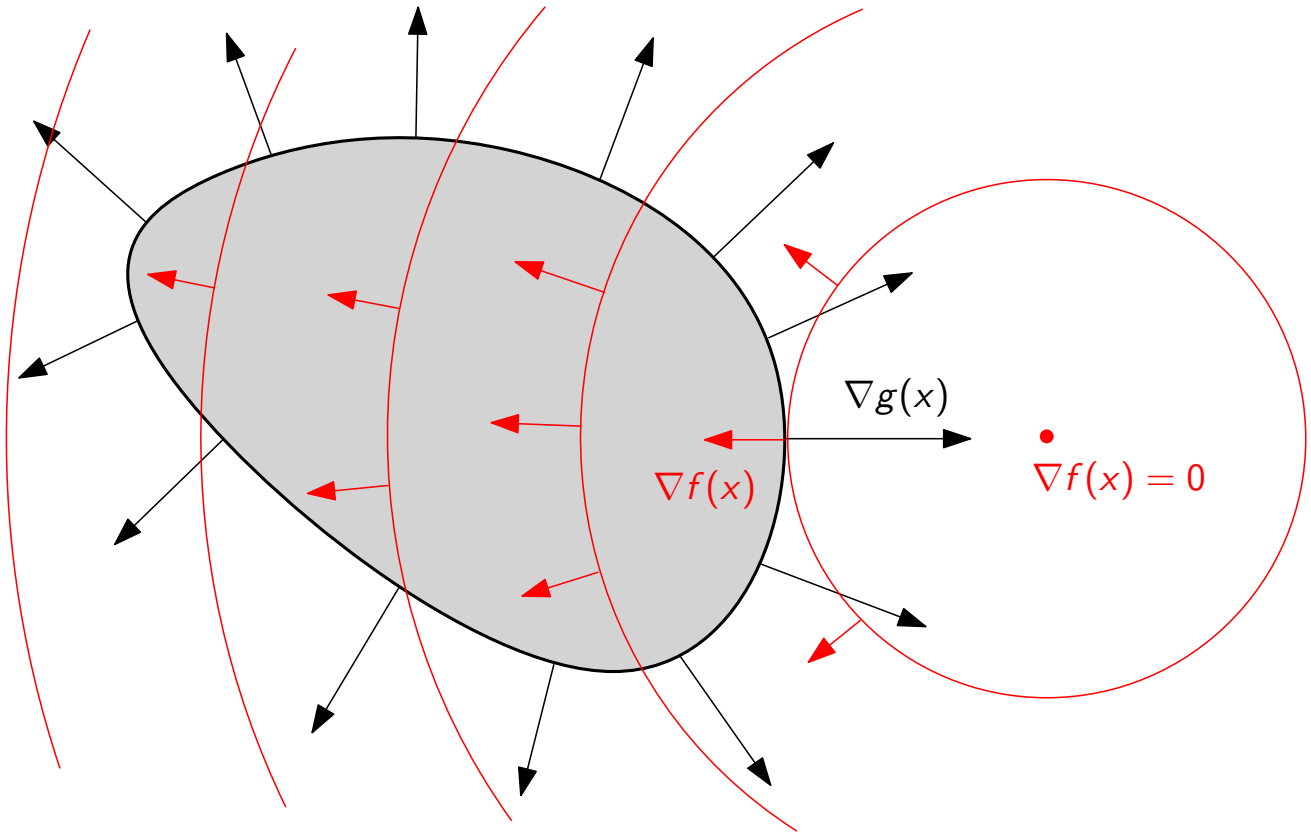
$$\nabla f(x^*)^\top (y - x^*) \geq 0 \qquad \text{for all feasible } y$$



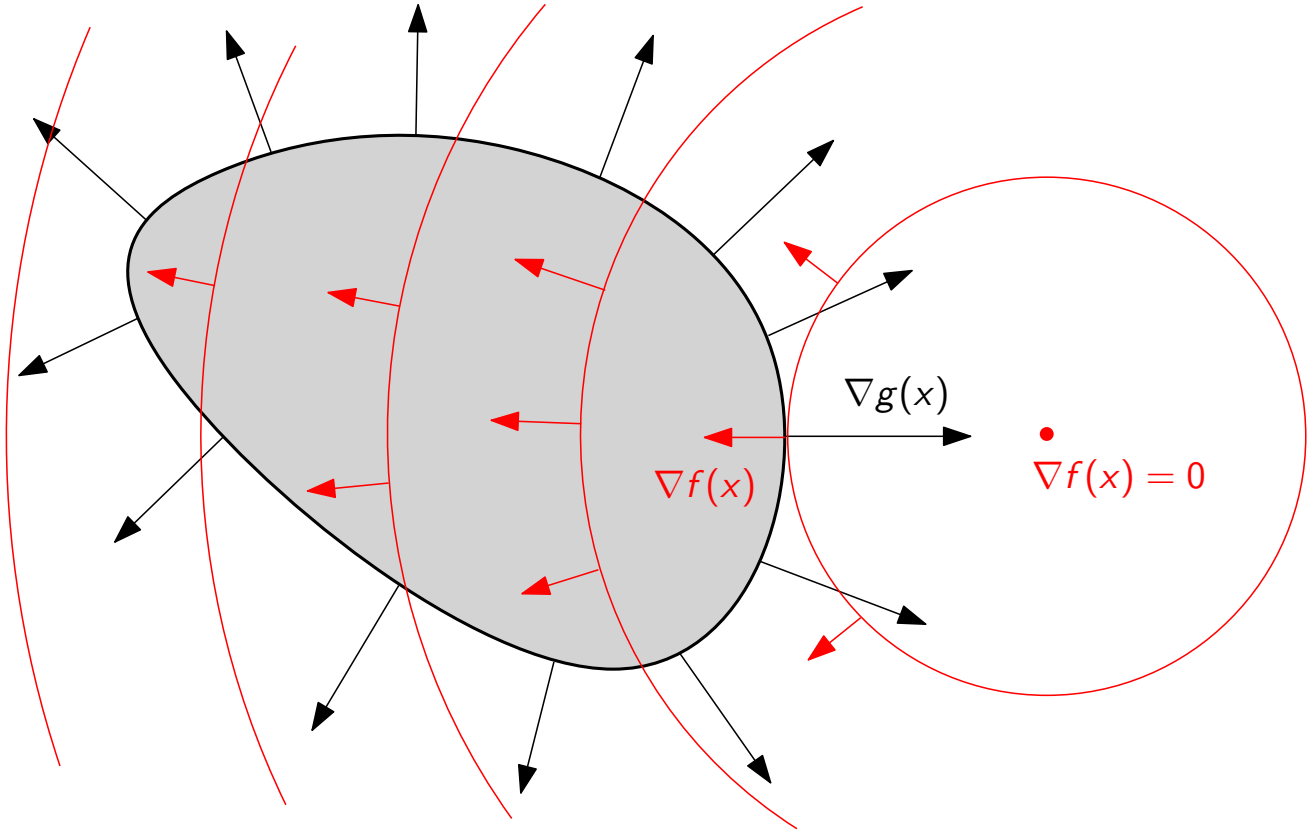
If the problem is unconstrained $\rightarrow$ $\nabla f(x^*) = 0$

Constrained case

$$\begin{aligned} \text{minimize}_{\,x\in\mathcal{X}} \quad & f(x) \\ \text{subject to} \quad & g(x) \leq \mathbf{0} \\ & h(x) = \mathbf{0}. \end{aligned}$$



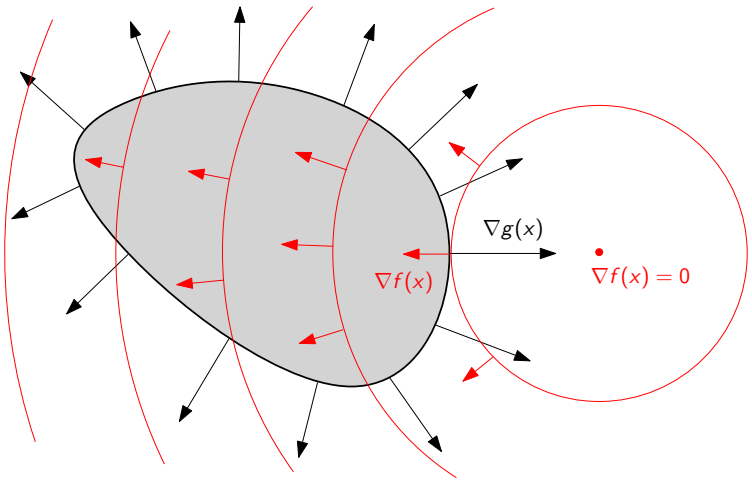
Constrained case



There exist* **multipliers** / **dual variables** μ such that

$$\begin{cases} \nabla f(x) + \mu^\top \nabla g(x) = 0 \qquad \mu \geq 0 \\ g(x) \leq 0 \\ \mu_i g_i(x) = 0 \quad \forall i \end{cases}$$

Solving convex optimization problems



- iterative gradient-based algorithms
- interior point algorithms

KKT conditions

$$\begin{cases} \nabla f(x) + \mu^\top \nabla g(x) = 0 \\ \mu \geq 0 \\ g(x) \leq 0 \\ \mu_i g_i(x) = 0 \quad \forall i \end{cases}$$

- iterative algorithms to solve (KKT)
- Lagrangian: function whose saddle-point solves (KKT)

Linear algebra + zero-finding = easy

Resources

`https://web.stanford.edu/~boyd/cvxbook/`

- Lecture slides
- Book “Convex Optimization” free to download
- Code and toolboxes (CVX)
- Exercises
- Open online course



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https://bsaver.io/COCO