

Power System Dynamic Modeling

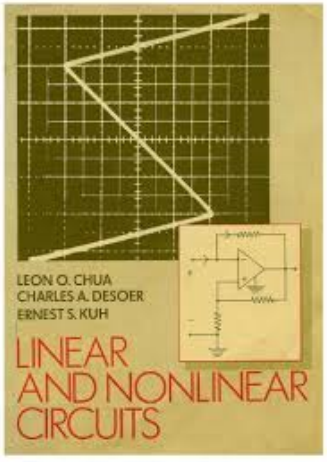
Florian Dörfler, ETH Zurich

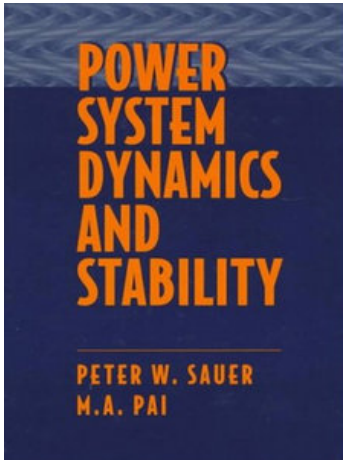
EECI School M19, June 20-23, 2022, Stockholm

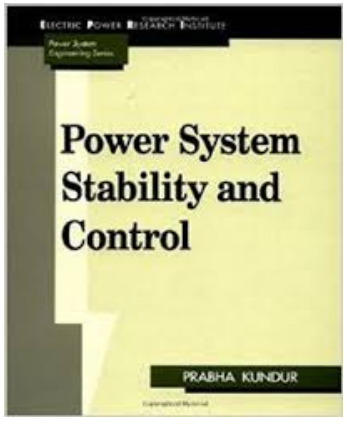
Outline

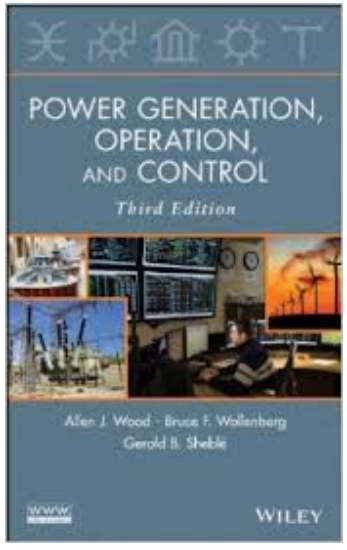
- Signal spaces & transforms
- Modeling of synchronous machines
- Reduced-order models & network models

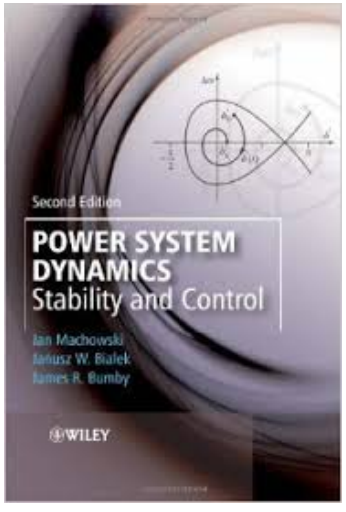
Many references available on power system modeling, stability, & control

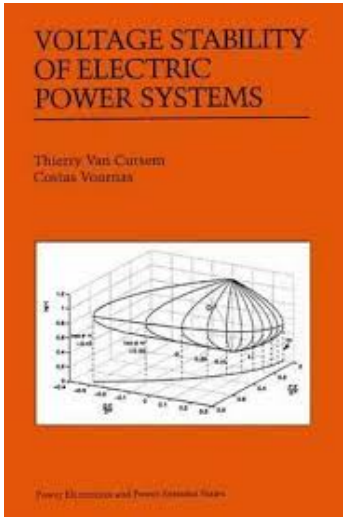


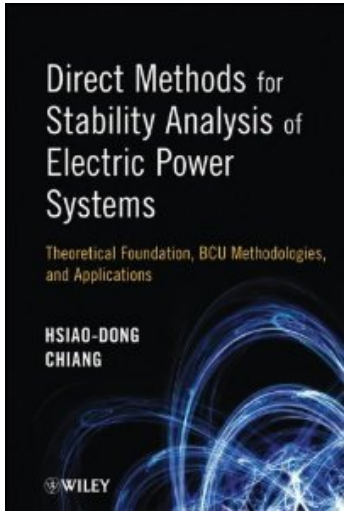


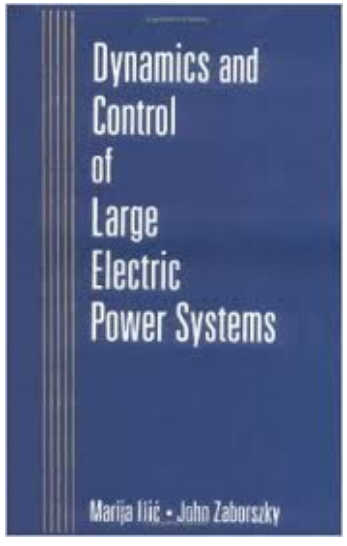




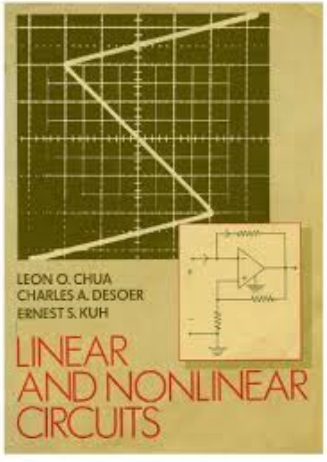


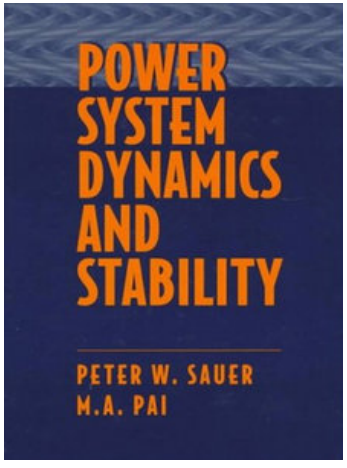


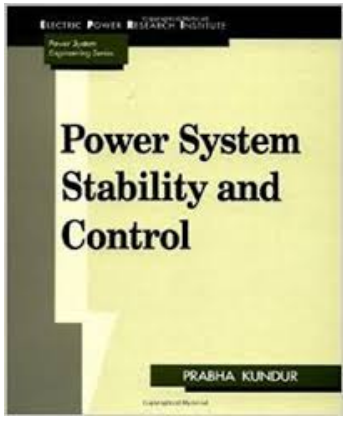


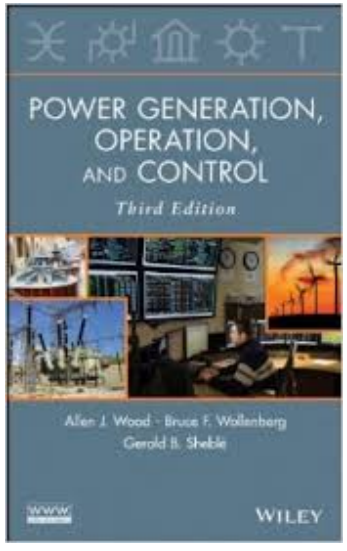


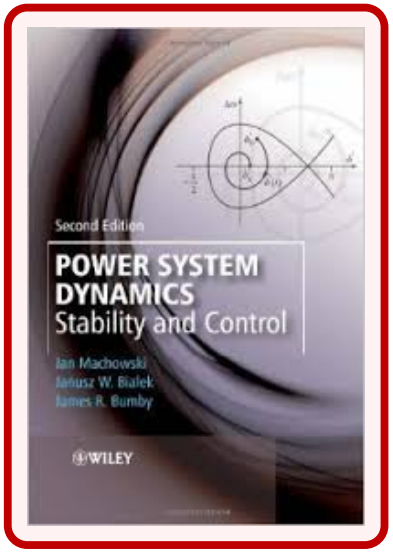
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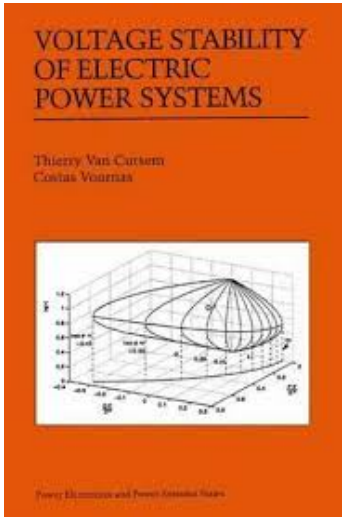


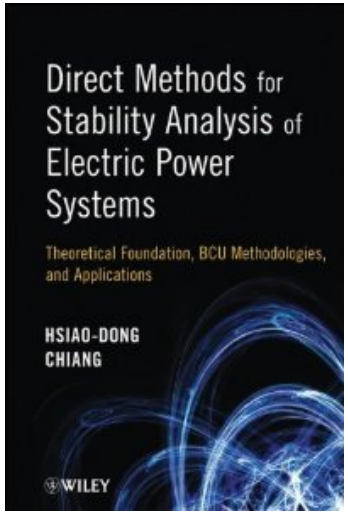


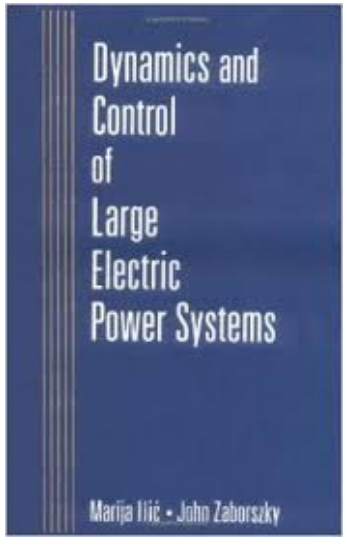












Main look-up reference for this part

up to some $\pm$ sign conventions

Control of Low-Inertia Power Systems

Florian Dörfler[1] and Dominic Groß[2]

[1]Automatic Control Laboratory, ETH Zurich, Zurich, Switzerland, 8092; email: dorfler@ethz.ch
[2]Electrical and Computer Engineering, University of Wisconsin-Madison, Madison, United States, WI 53706; email: dominic.gross@wisc.edu

Outline

- Signal spaces & transforms
- Modeling of synchronous machines
- Reduced-order models & network models

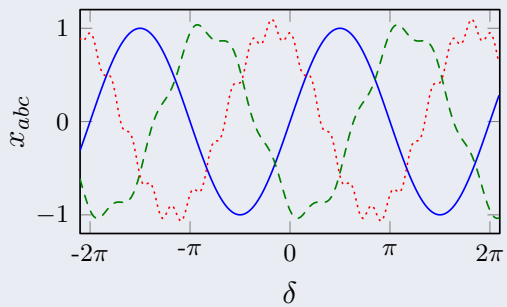
Modeling review: 3-phase signals

three-phase AC

$$\begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} = \begin{bmatrix} x_a(t+T) \\ x_b(t+T) \\ x_c(t+T) \end{bmatrix}$$

periodic with 0 average

$$\frac{1}{T}\int_0^T x_i(t)dt = 0$$



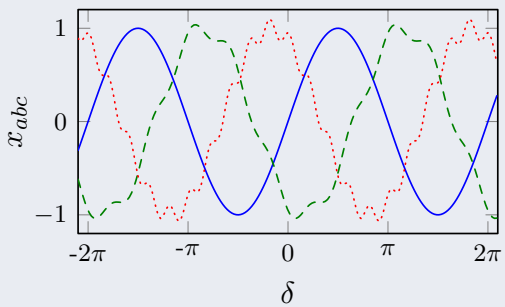
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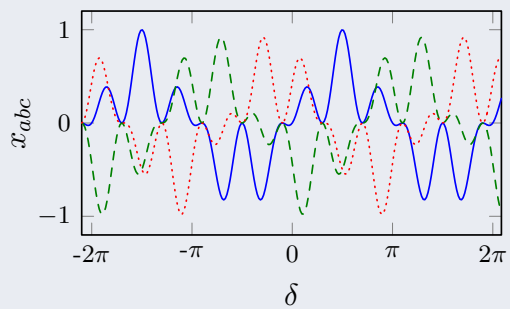


balanced (nearly true)

$$= A(t) \begin{bmatrix} \sin(\delta(t)) \\ \sin(\delta(t) - \frac{2\pi}{3}) \\ \sin(\delta(t) + \frac{2\pi}{3}) \end{bmatrix}$$

so that

$$x_a(t) + x_b(t) + x_c(t) = 0$$



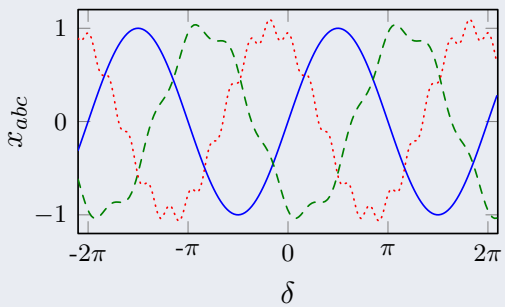
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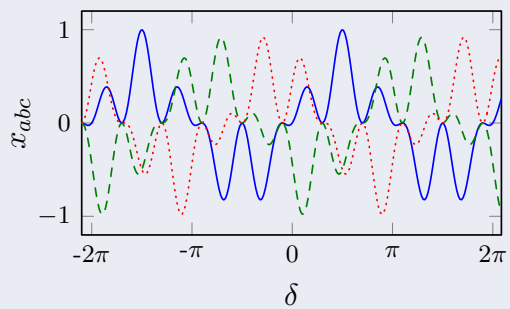


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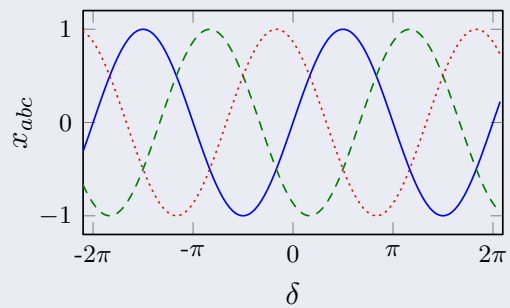


synchronous (desired)

$$= A \begin{bmatrix} \sin(\delta_0 + \omega_0 t) \\ \sin(\delta_0 + \omega_0 t - \frac{2\pi}{3}) \\ \sin(\delta_0 + \omega_0 t + \frac{2\pi}{3}) \end{bmatrix}$$

const. freq & amp

$\Rightarrow$ const. in rot. frame



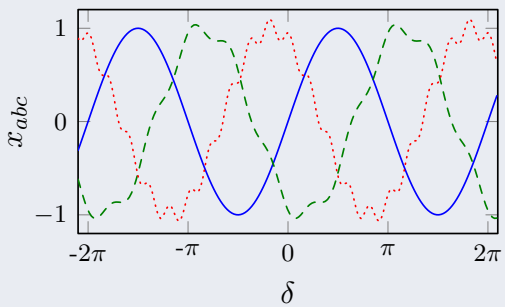
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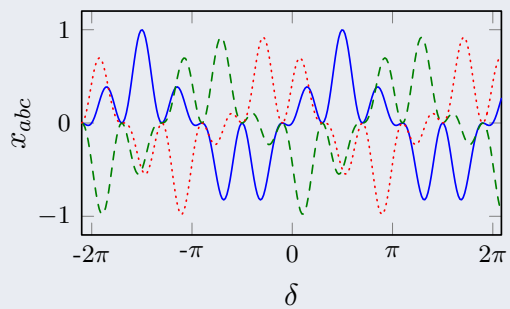


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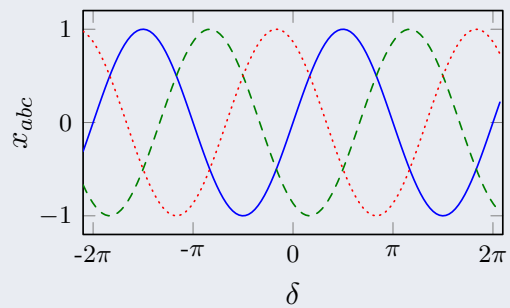


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const. freq & amp

$\Rightarrow$ const. in rot. frame



signal representations are over-parameterized $\rightarrow$ **Clarke / Park transforms**

Transforms of 3-phase balanced signals

$$x_{abc}$$

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} \sin(\delta) \\ \sin(\delta - \frac{2\pi}{3}) \\ \sin(\delta + \frac{2\pi}{3}) \end{bmatrix}$$

$\rightarrow$ orthogonal to $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

$$x_a(t) + x_b(t) + x_c(t) = 0$$

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orthonormal **Clarke transform**: $x_{abc} \rightarrow x_{\alpha\beta\gamma}$
removing the balanced subspace $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

$$T_{\alpha\beta\gamma} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \hline \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

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$x_{\alpha\beta\gamma} = T_{\alpha\beta\gamma}\, x_{abc}$

$$\left[\begin{array}{c} x_\alpha \\ x_\beta \\ \hline x_\gamma \end{array}\right] = \sqrt{\frac{3}{2}} \left[\begin{array}{c} \sin(\delta) \\ -\cos(\delta) \\ \hline 0 \end{array}\right]$$

$\rightarrow x_\gamma$ often discarded & $x_{\alpha\beta}$

shown as phasor $e^{\mathrm{i}\left(\delta - \frac{\pi}{2}\right)}$

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orthonormal **Clarke transform**: $x_{abc} \rightarrow x_{\alpha\beta\gamma}$ removing the balanced subspace $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

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orthonormal **Park transform**: $x_{\alpha\beta\gamma} \rightarrow x_{dq0}$ into rotating frame with angle θ

$$T_{dq0} = \sqrt{\tfrac{2}{3}} \left[\begin{array}{cc|c} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ \hline 0 & 0 & 1 \end{array}\right]$$

Transforms of 3-phase balanced signals

x_{abc}

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} \sin(\delta) \\ \sin(\delta - \frac{2\pi}{3}) \\ \sin(\delta + \frac{2\pi}{3}) \end{bmatrix}$$

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$x_{dq0} = T_{dq0}\, x_{\alpha\beta\gamma}$

$$\left[\begin{array}{c} x_d \\ x_q \\ \hline x_0 \end{array}\right] = \sqrt{\frac{3}{2}} \left[\begin{array}{c} \sin(\theta + \delta) \\ -\cos(\theta + \delta) \\ \hline 0 \end{array}\right]$$

$\rightarrow$ typical choice $\theta = -\delta$

$\theta = -\delta$

$\downarrow$

$= \sqrt{3/2} \begin{bmatrix} 0 \\ -1 \\ \hline 0 \end{bmatrix}$

Transforms of 3-phase balanced signals

x_{abc}

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} \sin(\delta) \\ \sin(\delta - \frac{2\pi}{3}) \\ \sin(\delta + \frac{2\pi}{3}) \end{bmatrix}$$

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$$\left[\begin{array}{c} x_d \\ x_q \\ \hline x_0 \end{array}\right] = \sqrt{\frac{3}{2}} \left[\begin{array}{c} \sin(\theta + \delta) \\ -\cos(\theta + \delta) \\ \hline 0 \end{array}\right]$$

$\rightarrow$ typical choice $\theta = -\delta$

$x_{dq0} = T_{dq0} \cdot T_{\alpha\beta\gamma}\, x_{abc}$ with overall transform

$$\sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos\left(\theta + \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) \\ \sin(\theta) & \sin\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

it’s tedious but useful to work through these calculations once in your lifetime

$\alpha\beta\gamma \to dq0$ & rotation matrix tricks

(covered on the board)

- $R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$, $R(-\theta) = R(\theta)^T$
- complex embedding: $R(\theta) \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \approx e^{i\theta}$

  $R(\theta) = \left[e^{i\theta} \mid e^{i(\theta - \pi/2)} \right]$
- $R(\theta) \cdot R(s) = R(\theta + s)$
- inverse $R(\theta) \cdot R(-\theta) = I \Rightarrow R(\theta)^{-1} = R(-\theta) = R(\theta)^T$
- "imaginary unit" $J = \begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix}$, $J \cdot J = -I$, $J^T = -J$

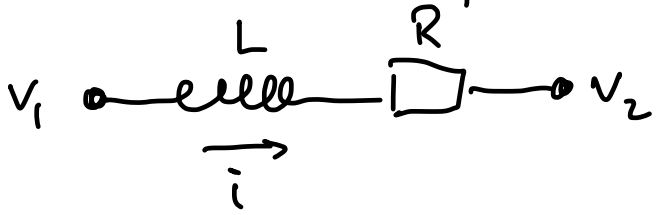
  $\hat{=} e^{i\pi/2}$ $\hat{=} R(\pi/2)$ $i^2 = -1$ $\bar{i} = -i$
- $\frac{d}{dt} R(\theta(t)) = J \cdot \dot{\theta} \cdot R(\theta(t))$

  $d/dt\ e^{i\theta(t)} = i\dot{\theta} e^{i\theta(t)}$

$\frac{d}{dt} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} -s & -c \\ c & -s \end{bmatrix}$

$\hat{=} J \cdot R(\theta)$

Application: dq transform of circuit equations:



$L, R > 0$ (scalars)

$V_1, V_2 \in \mathbb{R}^2$: $\alpha\beta$ voltages

- $\alpha\beta$: $\frac{d}{dt} L i = -R i + V_1 - V_2$

- dq coordinates with const. frequency $\omega \in \mathbb{R}$

$I = R(-\omega t) i$, $V_k = R(-\omega t)\, v_k$ $(k \in \{1,2\})$

$$\frac{d}{dt} L I = L \frac{d}{dt} \left(R(-\omega t) \cdot i\right) = L\, R(-\omega t) \cdot \frac{d}{dt} i + L \frac{d}{dt} \left(R(-\omega t)\right) \cdot i$$

$$= -\omega \cdot J L\, R(-\omega t) \cdot i - R(-\omega t) \cdot R i + V_1 - V_2$$

$$= -\omega J L \cdot I - R I + V_1 - V_2$$

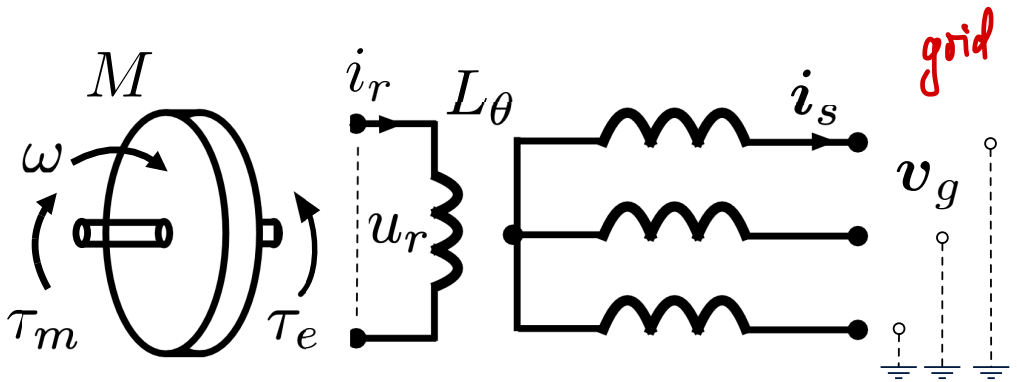
$$d/dt\, L I = -(R + J\omega L) I + V_1 - V_2$$

Outline

- Signal spaces & transforms
- Modeling of synchronous machines
- Reduced-order models & network models

Synchronous machine modeling

(assumptions: non-salient rotor, single-pole pair, dc-excitation, no damper windings)



mechanical equations

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega$$

$$M\frac{\mathrm{d}\omega}{\mathrm{d}t} = -D\omega + \tau_m - \tau_e$$

flux linkage equations

abc →

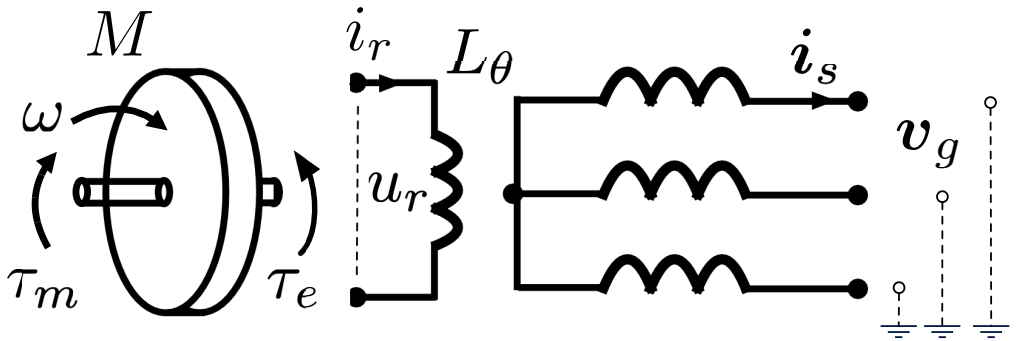
$$\frac{\mathrm{d}\boldsymbol{\lambda}_s}{\mathrm{d}t} = -R_s\boldsymbol{i}_s + \boldsymbol{v}_g$$

scalar →

$$\frac{\mathrm{d}\lambda_r}{\mathrm{d}t} = -R_r i_r + u_r$$

Synchronous machine modeling

(assumptions: non-salient rotor, single-pole pair, dc-excitation, no damper windings)



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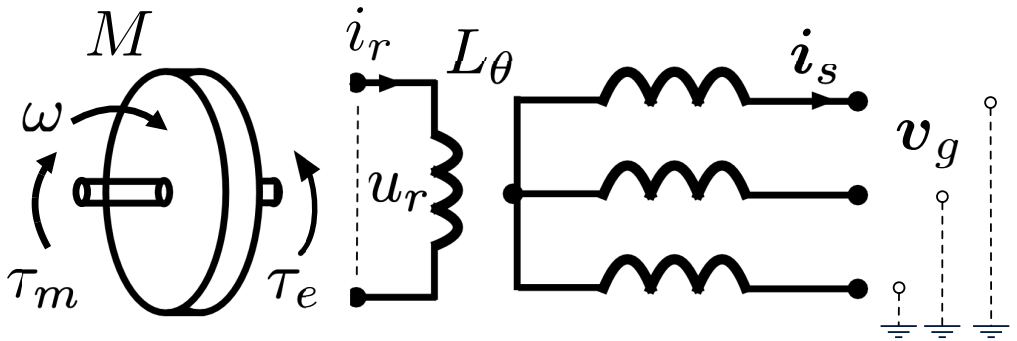
inductance matrix for $\boldsymbol{\lambda}_s$ (abc) & λ_r (scalar)

$$L_\theta = \begin{bmatrix} L_s & 0 & 0 & L_m\cos(\theta) \\ 0 & L_s & 0 & L_m\cos(\theta-\frac{2\pi}{3}) \\ 0 & 0 & L_s & L_m\cos(\theta+\frac{2\pi}{3}) \\ L_m\cos(\theta) & L_m\cos(\theta-\frac{2\pi}{3}) & L_m\cos(\theta+\frac{2\pi}{3}) & L_r \end{bmatrix}$$

$$L(\theta)\cdot\begin{bmatrix} i_s \\ i_r \end{bmatrix} = \begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix}$$

Synchronous machine modeling

(assumptions: non-salient rotor, single-pole pair, dc-excitation, no damper windings)



mechanical equations

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electro-magnetic energy

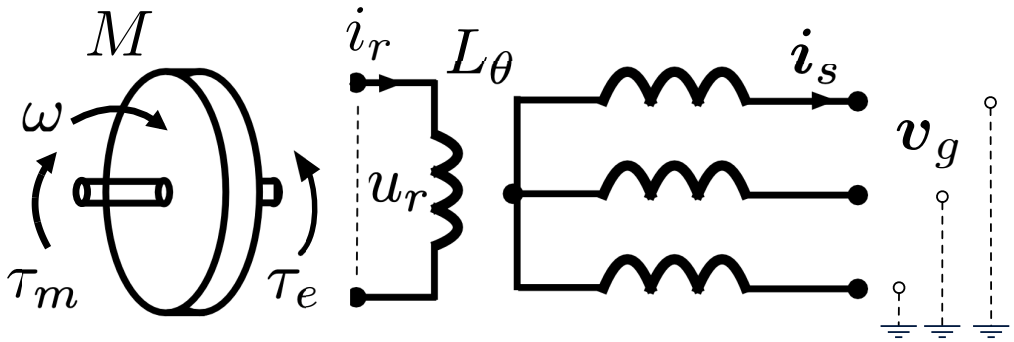
$$W = \frac{1}{2}\begin{bmatrix}\boldsymbol{\lambda}_s \\ \lambda_r\end{bmatrix}^\top L_\theta^{-1}\begin{bmatrix}\boldsymbol{\lambda}_s \\ \lambda_r\end{bmatrix} = \frac{1}{2}\begin{bmatrix}\boldsymbol{i}_s \\ i_r\end{bmatrix}^\top L(\theta)\cdot\begin{bmatrix}\boldsymbol{i}_s \\ i_r\end{bmatrix}$$

air gap torque

$$\tau_e = \frac{\partial W}{\partial \theta}$$

Synchronous machine modeling

(assumptions: non-salient rotor, single-pole pair, dc-excitation, no damper windings)



mechanical equations

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega$$

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electro-magnetic energy

$$W = \tfrac{1}{2}\begin{bmatrix}\boldsymbol{\lambda}_s \\ \lambda_r\end{bmatrix}^\top L_\theta^{-1}\begin{bmatrix}\boldsymbol{\lambda}_s \\ \lambda_r\end{bmatrix}$$

air gap torque

$$\tau_e = \frac{\partial W}{\partial\theta}$$

stator / rotor currents

$$\boldsymbol{i}_s = \frac{\partial W}{\partial\boldsymbol{\lambda}_s}^\top \Leftrightarrow \boldsymbol{\lambda}_s = L_s\boldsymbol{i}_s + L_m\begin{bmatrix}\cos(\theta)\\ \cos(\theta-\frac{2\pi}{3})\\ \cos(\theta+\frac{2\pi}{3})\end{bmatrix} i_r$$

$$i_r = \frac{\partial W}{\partial\lambda_r} \Leftrightarrow \lambda_r = L_r i_r + L_m\begin{bmatrix}\cos(\theta)\\ \cos(\theta-\frac{2\pi}{3})\\ \cos(\theta+\frac{2\pi}{3})\end{bmatrix}^\top \boldsymbol{i}_s$$

$\lambda = L(\theta)\cdot i$

Port-Hamiltonian system modeling

(covered on the board)

abstract:

$$\dot{x} = \big(J(x) - R(x) \big) \cdot \nabla H(x) + G(x) \cdot u$$

$J(x) = -J^T(x)$ — skew-symmetric "interconnected"

$R(x) = R^T(x) \geq 0$ — dissipation

H — Hamiltonian / energy

power balance:

$$\frac{d}{dt} H(x) = \nabla H^T \cdot \dot{x} = \nabla H^T J \nabla H - \nabla H^T R \cdot \nabla H + \nabla H^T G \cdot u$$

$\nabla H^T J \nabla H = 0$

$-\nabla H^T R \cdot \nabla H \leq 0$

$\nabla H^T G = y(x)$

$+ \nabla H^T G \cdot u$ — external power supply

RL:

$$\frac{d}{dt} L I = -(R + j\omega L) I + \underbrace{V_1 - V_2}_{=u}$$

$$\lambda = +L I, \quad H = \frac{1}{2} L I^2 = \frac{1}{2} L^{-1} \lambda^2$$

$$\Rightarrow \frac{d}{dt} \lambda = \underbrace{(-j\omega - R I_2)}_{\text{skew-symmetric}} \cdot \frac{\partial}{\partial \lambda} H + u$$

Synchronous machine:

$$\begin{cases} \text{flux: } \lambda := L(\theta)\cdot i \\ \text{impulse: } p := \Pi\omega \end{cases} \qquad \begin{cases} \text{potential energy: } W = \frac{1}{2} i^T L(\theta)\cdot i = \frac{1}{2}\lambda^T \cdot L(\theta)^{-1}\lambda \\ \text{kinetic energy: } T = \frac{1}{2}\Pi\omega^2 = \frac{1}{2}\Pi^{-1}p^2 \end{cases}$$

$\Rightarrow$ **Hamiltonian:** $H = T(p) + W(\lambda,\theta)$

$$= \frac{1}{2}\Pi^{-1}p^2 + \frac{1}{2}\lambda^T \cdot L(\theta)^{-1}\lambda$$

$\rightarrow$ frequency: $\omega = \Pi^{-1}p = \nabla_p H$

current: $i = L(\theta)^{-1}\lambda = \nabla_\lambda H$

air-gap torque: $\tau_e = \frac{\partial}{\partial \text{position}}\{\text{potential energy}\} = \nabla_\theta W$

dynamics: $\dot\theta = \omega = \nabla_p H$

$$\Pi\dot\omega = -D\omega + \tau_m - \tau_e \Leftrightarrow \dot p = -D\cdot\nabla_p H + \tau_m - \nabla_\theta H$$

$$\begin{bmatrix} \dot\lambda_s \\ \dot\lambda_r \end{bmatrix} = \begin{bmatrix} -R_s & \\ & -R_r \end{bmatrix}\begin{bmatrix} i_s \\ i_r \end{bmatrix} + \begin{bmatrix} v_s \\ v_r \end{bmatrix}$$

all together:

$$\frac{d}{dt}\begin{bmatrix} \theta \\ p \\ \lambda \end{bmatrix} = \left(\begin{bmatrix} 0 & I & 0 \\ -I & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & +D & 0 \\ 0 & 0 & +R \end{bmatrix} \right) \cdot \begin{bmatrix} \nabla_\theta H \\ \nabla_p H \\ \nabla_\lambda H \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \tau_m \\ \hline v_g \\ u \end{bmatrix}}_{\hat{u}}$$

$$\begin{bmatrix} R_s & \\ & R_r \end{bmatrix}$$

power balance:

$$\dot{H} = \underbrace{-\nabla_p H \, D \, \nabla_p H}_{D\omega^2} - i^T \begin{bmatrix} R_s & \\ & R_r \end{bmatrix} i$$

$$+ \nabla H^T \cdot \hat{u} \quad \begin{cases} \omega \cdot \tau_m + i_s^T v_g + i_r u \\ \nabla H_p \cdot \tau_m + \nabla_\lambda H \cdot \begin{bmatrix} v_g \\ u \end{bmatrix} \end{cases}$$

Initial condition for further reading

Perspectives in modeling for control of power networks

Arjan van der Schaft[a,*], Tjerk Stegink[a,b,c]

[a] *Jan C. Willems Center for Systems and Control, University of Groningen, The Netherlands*
[b] *Johann Bernoulli Institute for Mathematics and Computer Science, University of Groningen, The Netherlands*
[c] *Engineering and Technology Institute Groningen, University of Groningen, The Netherlands*

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Networks

ABSTRACT

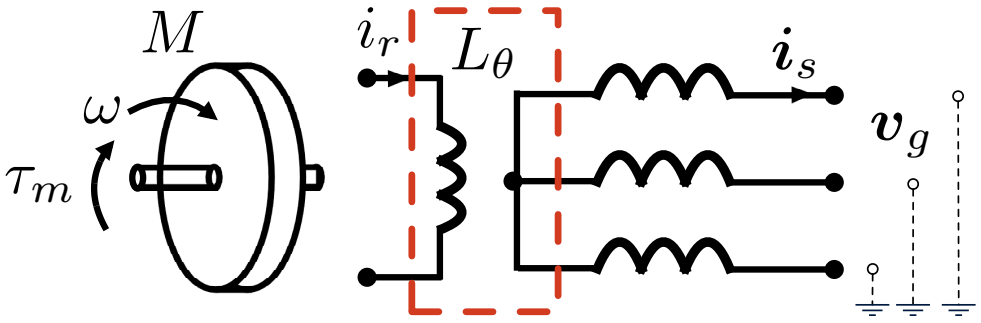
Increasing integration of renewable energy sources in the power grid leads to a complete re-thinking of its operation. This necessitates the consideration of new modeling and analysis approaches, which can serve as natural starting points for robust and scalable control and design strategies.

In this 'vision' paper we will highlight two topics within the broad area of power networks: the modeling and analysis of the synchronous generator, and the modeling and analysis of power networks using the swing equation as an approximate model for the generator. In both cases we will discuss a port-Hamiltonian formulation, which reflects the underlying physics of power flow and energy storage. Although the port-Hamiltonian model of the synchronous generator reveals a clear structure it still poses fundamental challenges for its non-zero steady state stability analysis. It is shown how the swing equation can be directly deduced from the power balance of the port-Hamiltonian model of the synchronous generator. Under the phasor assumption, this leads to a port-Hamiltonian model of power networks of generators, which enables a straightforward stability analysis and provides a starting point for control.

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putting all equations together &
converting to $\alpha\beta$ frame & currents

Synchronous generator in $\alpha\beta$ frame



$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega$$

$$M\frac{\mathrm{d}\omega}{\mathrm{d}t} = -D\omega + \tau_m + L_\mathsf{m} i_r \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}^\top \boldsymbol{i}_s$$

$$L_\mathsf{s}\frac{\mathrm{d}\boldsymbol{i}_s}{\mathrm{d}t} = -R_s\boldsymbol{i}_s + \boldsymbol{v}_g - L_\mathsf{m} i_r \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}\omega$$

input

Note: neglected i_r rotor current dynamics (assume tight control) $\rightarrow$ drop $\frac{d}{dt}i_r$ terms

L fast

1. **primary energy supply** τ_m from turbine converting thermal to mechanical energy (neglected)
2. mechanical (θ, ω) **swing dynamics** of rotor (flywheel) with inertia M
3. **electro-mechanical energy conversion** through rotating magnetic field with inductance matrix

$$L_\theta = \begin{bmatrix} L_\mathsf{s} & 0 & L_\mathsf{m}\cos\theta \\ 0 & L_\mathsf{s} & L_\mathsf{m}\sin\theta \\ L_\mathsf{m}\cos\theta & L_\mathsf{m}\sin\theta & L_\mathsf{r} \end{bmatrix}$$

4. $\boldsymbol{i}_s$ **stator flux dynamics** (sometimes including additional damper windings)
5. connection to grid with voltage $\boldsymbol{v}_g$

Conversion to machine angle dq frame

(covered on the board)

$\alpha\beta$ - dynamics:

$$\dot{\theta} = \omega$$

$$J\dot{\omega} = -D\omega + \tau_m + L_m i_r \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} i_s$$

$$L_s \dot{i}_s = -R_s i_s + v_s - L_m i_r \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \omega$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \cdot R(\theta)$$

$$\hookleftarrow R(\theta) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

dq: $\begin{bmatrix} i_d \\ i_q \end{bmatrix} = R(-\theta)\, i_s$

$$\frac{d}{dt}\begin{bmatrix} i_d \\ i_q \end{bmatrix} = -J\omega \underbrace{R(\theta) \cdot i_s}_{\begin{bmatrix} i_d \\ i_q \end{bmatrix}} + R(-\theta) \cdot \frac{d}{dt} i_s$$

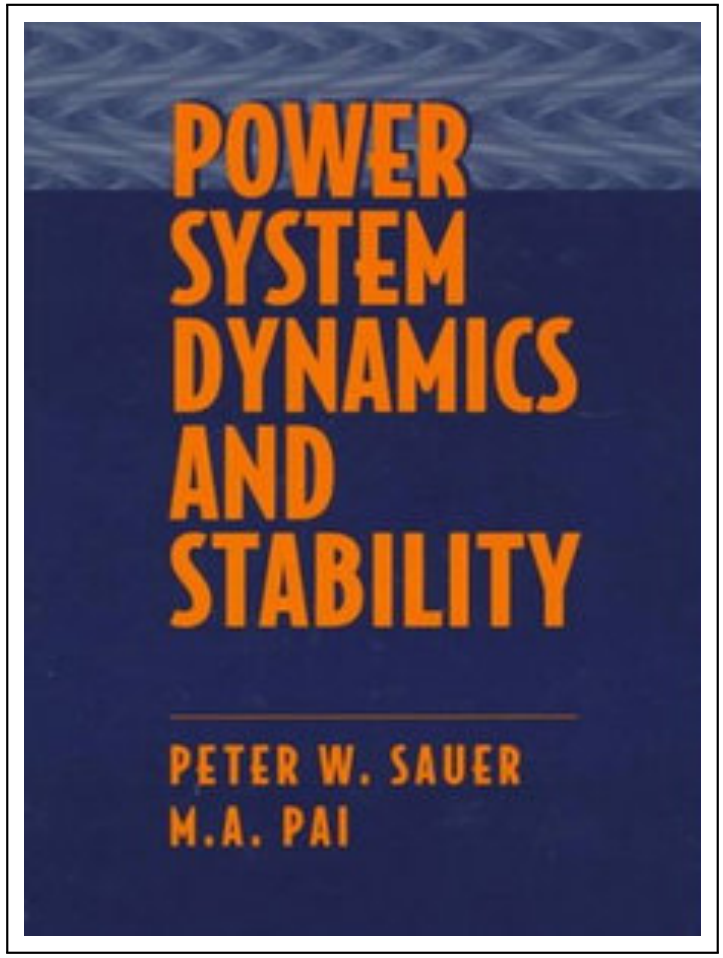
$\vdots$

at home

Outline

- Signal spaces & transforms
- Modeling of synchronous machines
- Reduced-order models & network models

Power systems involves some science, engineering, & *“art”* — model reduction



This book makes the “art” more scientific.

Derivation of the swing equation

(covered on the board)

power balance of a synchronous machine

$$\frac{d}{dt}\left[\frac{1}{2}\Pi\omega^2 + \frac{1}{2} i^T L(\theta)\cdot i\right]$$

$\Rightarrow$ neglect (on longer time scales $> 3{,}4\,s$)

$$= \omega\Pi\dot{\omega} + 0$$

$$= -D\omega^2 + \omega\cdot\tau_m - i^T R i + v_g^T i_s + v^T i_r$$

($D\omega^2 \approx 0$, $i^T R i \approx 0$, $v^T i_r = 0$)

$$= P_m - P_e = \text{mechanical power} - \text{electrical power}$$

(P_m defined, $\hat{=}\ \tilde{P}_m$; $P_e \hat{=}\ \tilde{P}_e$)

$$\Rightarrow \frac{\omega\,\Pi\dot{\omega}}{\text{base}} = \frac{P_m}{\text{base}} - \frac{P_e}{\text{base}} \leftarrow \text{normalization}$$

$$= \frac{\frac{1}{2}\Pi\omega^2}{2\cdot\text{base}\cdot\omega}\cdot\dot{\omega} \quad (= H = \text{const.})$$

assumption

$$H\dot{\omega} = \tilde{P}_m - \tilde{P}_e$$

$$H_i \dot{\omega}_i = P_{m,i} - P_{e,i} \qquad V_i = |V_i| e^{j\theta_i}$$

$$i_i = \sum_k Y_{ik} \cdot |V_k| e^{j\theta_k}$$

$$= P_{m,i} - \mathrm{Re}\left(-V_i^T i_i\right)$$

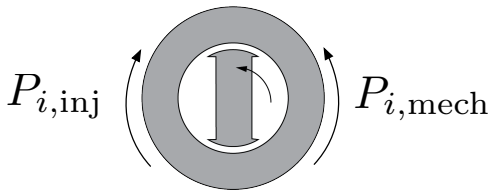
$$= P_{m,i} - \sum_{j=1}^{\cdots} B_{ij} |V_i| \cdot |V_j| \sin(\theta_i - \theta_j)$$

↑ grid lossless

$$\frac{\partial}{\partial \theta} \sum_j \cdots \cos(\theta_i - \theta_j)$$

Modeling the “essential” generator dynamics

1. electromech. **swing dynamics** of synchronous machines



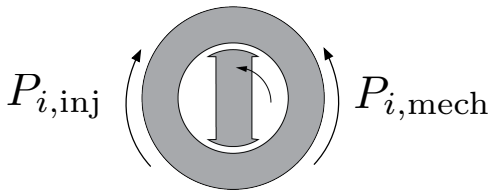
$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_{i,\text{mech}} - P_{i,\text{inj}}$$

const. terminal voltage magnitude E_i.

(can be derived from first principle model & some (possibly strong) assumptions)

Modeling the “essential” generator dynamics

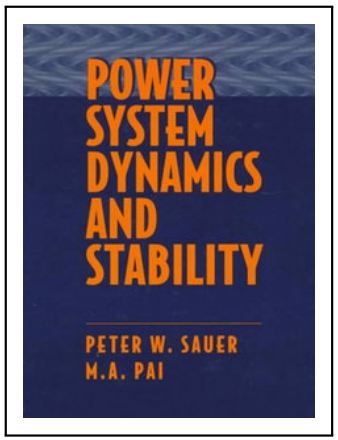
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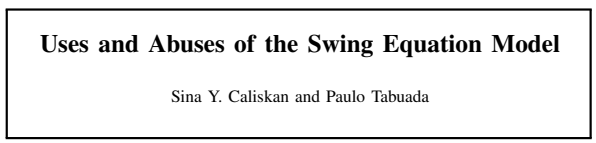
const. terminal voltage magnitude E_i.

(can be derived from first principle model & some (possibly strong) assumptions)



“There is probably more literature on synchronous machines than on any other device in electrical engineering.”

— [P. Sauer & M.A. Pai, Power System Dynamics and Stability ’98]



“The swing equation model is a perfect example of the famous line [. . .]: “All models are wrong, but some are useful.””

— [S.Y. Caliskan and P. Tabuada, CDC ’15]

Modeling the “essential” network dynamics

models can be arbitrarily detailed & vary on different time/spatial scales

2. active and reactive **power flow**

   (≈ lossless for high voltage)

$$P_{i,\text{inj}} = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

$$Q_{i,\text{inj}} = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

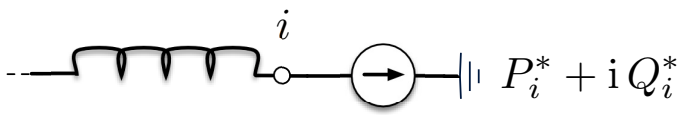
Modeling the “essential” network dynamics

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3. passive constant power **loads**



$$P_{i,\text{inj}} = P_i = const.$$
$$Q_{i,\text{inj}} = Q_i = const.$$

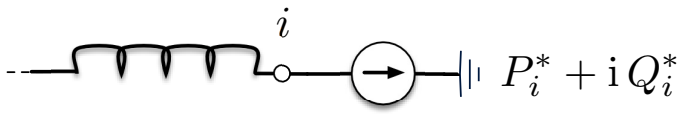
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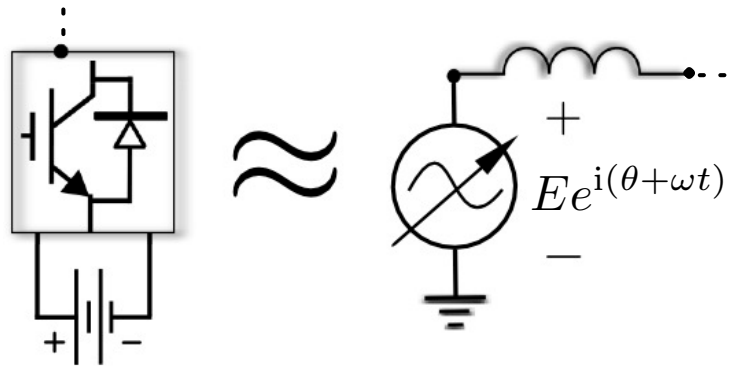
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3. passive constant power **loads**



$$P_{i,\text{inj}} = P_i = const.$$
$$Q_{i,\text{inj}} = Q_i = const.$$

4. **inverters**: DC or variable AC sources with power electronics



(i) have constant/controllable PQ
(max. power-point tracking)

(ii) or mimic generators
(many more details later)

Structure-preserving power network model

without Kron-reduction of load buses [A. Bergen & D. Hill '81]

- **generator swing dynamics:**

$$\dot{\theta}_i = \omega_i$$

$$M_i\dot{\omega}_i = -D_i\omega_i + P_i - \sum\nolimits_j B_{ij}E_iE_j\sin(\theta_i - \theta_j)$$

$$Q_i = -\sum\nolimits_j B_{ij}E_iE_j\cos(\theta_i - \theta_j)$$

- **frequency-dependent loads:**
  (or inverter-interfaced sources)

$$D_i\dot{\theta}_i = P_i - \sum\nolimits_j B_{ij}E_iE_j\sin(\theta_i - \theta_j)$$

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- in **academia:** this "baseline model" is typically further simplified: decoupling, linearization, constant voltages, ...
- in **industry:** much more detailed models used for grid simulations

$\Rightarrow$ **IMHO:** above model captures most interesting network dynamics

Common variations in dynamic network models

dynamic behavior is very much dependent on load models & generator models

1. frequency/voltage-depend. loads

   [A. Bergen & D. Hill '81, I. Hiskens & D. Hill '89, R. Davy & I. Hiskens '97]

$$D_i\dot{\theta}_i + P_i = -P_{i,\text{inj}}$$

$$f_i(\dot{V}_i) + Q_i = -Q_{i,\text{inj}}$$

Common variations in dynamic network models

dynamic behavior is very much dependent on load models & generator models

1. frequency/voltage-depend. loads
   [A. Bergen & D. Hill '81, I. Hiskens & D. Hill '89, R. Davy & I. Hiskens '97]

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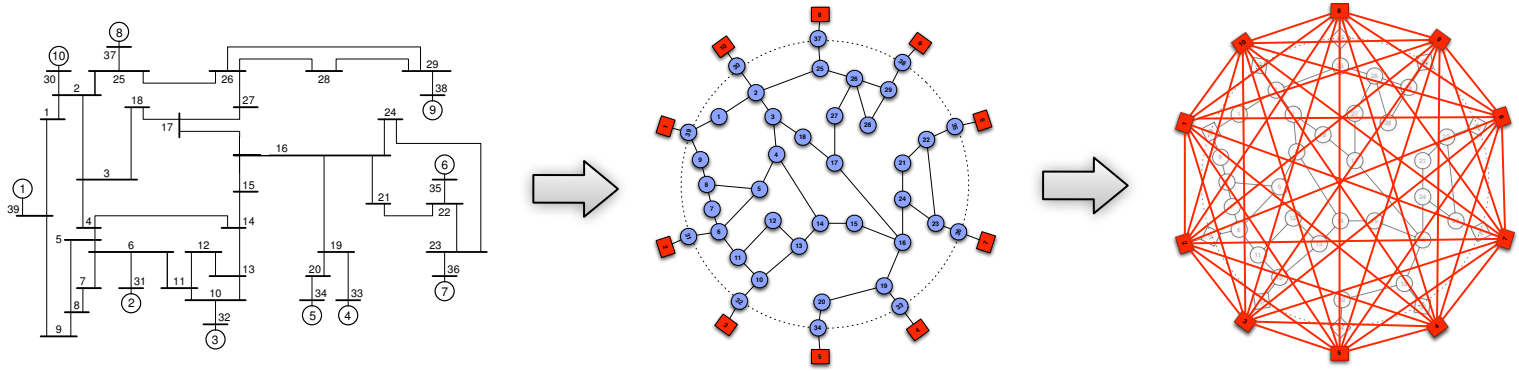
2. network-reduced models after Kron reduction of loads
   [H. Chiang, F. Wu, & P. Varaiya '94]
   (common but poor assumption: $G_{ij} = 0$)

generator + reduced load

$$M_i\ddot{\theta}_i + D\dot{\theta}_i = P_{i,\text{mech}} \underbrace{- \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) - \sum_j G_{ij} E_i E_j \cos(\theta_i - \theta_j)}_{\text{effect of resistive loads}}$$

$\frac{\partial}{\partial\theta} U(\theta)$

non-potential



Derivation of network-reduced model

(covered on the board)

node dynamics:

- generators: $\dot{\theta}_i = \omega_i$

  $M_i \dot{\omega}_i = -D_i \omega_i + P_{m,i} - P_{e,i}$

  (leave out reactive power) $\quad P_{e,i} = \mathrm{Re}(V_{gen,i} \cdot i_{gen,i}^*)$

- loads: ~~Z~~ I - model

network: $I = (Y + Y_{shunt})\, V$

$$\begin{bmatrix} I_{gen} \\ I_{load} \end{bmatrix} = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{12}^* & Y_{22} \end{array}\right] + \begin{bmatrix} 0 & \\ & Y_{shunt} \end{bmatrix} \begin{bmatrix} V_{gen} \\ V_{load} \end{bmatrix}$$

lossless — resistive (dominantly)

Kron-reduction: eliminate $V_{load} = (Y_{22} + Y_{shunt})^{-1} (I_{load} - Y_{12}^* V_{gen})$

$$\Rightarrow \quad \underbrace{I_{gen} - Y_{12} (Y_{22} + Y_{shunt})^{-1} I_{load}}_{\tilde{I}_{gen}} = \underbrace{\left[Y_{11} - Y_{12} (\dots)^{-1} Y_{12}^*\right]}_{Y_{red} \approx \text{inductive + resistive}} V_{gen}$$

reduced generator dynamics:

$$\dot{\theta}_i = \omega_i$$

$$\Pi_i \dot{\omega}_i = -D_i \omega_i + \underbrace{p_{m,i} - \mathrm{Re}\left(V_i \cdot \tilde{\tilde{I}}_{gen,i}\right)}$$

$\hat{=}\ \hat{p}_{m,i}$ = net power injection

$$- \sum_j B_{ij} \sin(\theta_i - \theta_j) + G_{ij} \cos(\theta_i - \theta_j)$$

B_{ij}: imaginary, G_{ij}: real

elements of Y_{red}

Common variations in dynamic network models cont'd

dynamic behavior is very much dependent on load models & generator models

3. higher order generator dynamics
   [P. Sauer & M. Pai '98]

   voltages, controls, magnetics etc.
   (reduction via singular perturbations)

4. dynamic & detailed load models
   [D. Karlsson & D. Hill '94]

   aggregated dynamic load behavior
   (e.g., load recovery after voltage step)

5. time-domain models
   [S. Caliskan & P. Tabuada '14, S. Fiaz et al. '12]

   passive Port-Hamiltonian models
   for machines & RLC circuitry

Common variations in dynamic network models cont'd

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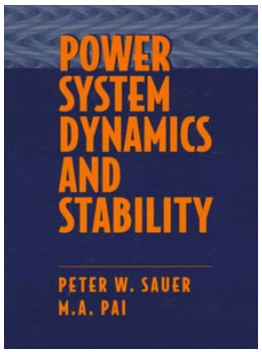
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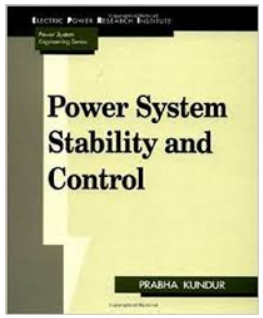
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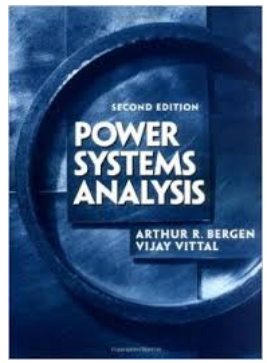
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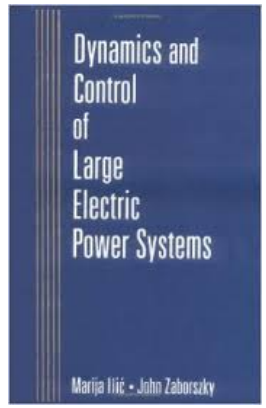
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   [S. Caliskan & P. Tabuada '14, S. Fiaz et al. '12]

   passive Port-Hamiltonian models for machines & RLC circuitry









"Power system research is all about the art of making the right assumptions."

Port-Hamiltonian representation of network-reduced swing equation model

(covered on the board)

$$\dot{\theta}_i = \omega_i$$

$$\Pi_i \dot{\omega}_i = -D_i \omega_i + p_{m,i} - \underbrace{\sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)}_{\text{with } E_i = |V_i| = 1}$$

$$\omega_i = \Pi_i^{-1} p_i \,, \quad T = \frac{1}{2} p^T \Pi^{-1} p \qquad \frac{\partial}{\partial \theta_i} U(\theta)$$

$$U(\theta) = -\sum_i \sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + \sum_i p_{m,i} \cdot \theta_i$$

$$\hookrightarrow \begin{bmatrix} \dot{\theta} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -I & -D \end{bmatrix} \begin{bmatrix} \nabla H_p \\ \nabla H_\theta \end{bmatrix} + \begin{bmatrix} 0 \\ p_m \end{bmatrix}$$

$= D\omega^2$

$$\hookrightarrow \text{power balance: } \dot{H} = -\nabla_p H^T D \nabla_p H - \nabla_\theta H \cdot p_m$$

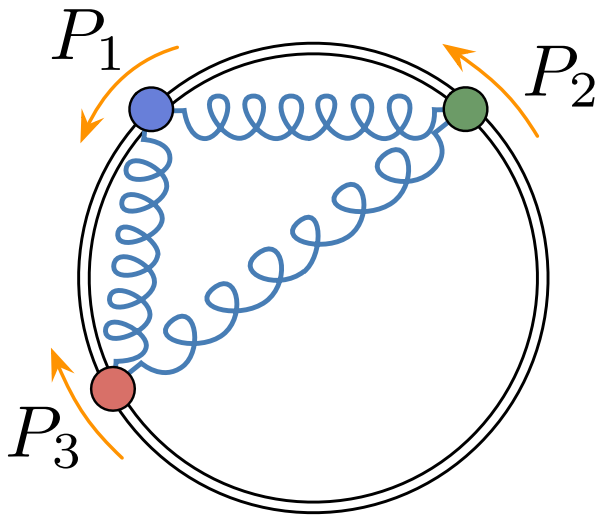
A coupled oscillator analogy

Mechanical oscillator network

Angles $(\theta_1, \ldots, \theta_n)$ evolve on $\mathbb{T}^n$ as

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$

- inertia constants $M_i > 0$
- viscous damping $D_i > 0$
- external torques $P_i \in \mathbb{R}$
- spring constants $B_{ij} \geq 0$



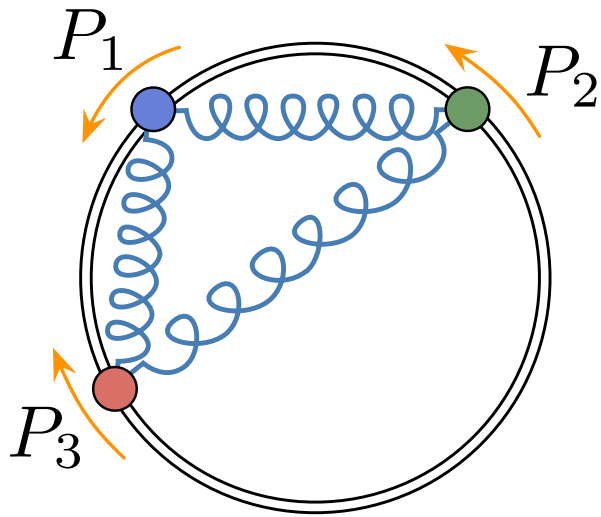
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Structure-preserving power network

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$
$$D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$

