

Grid-Forming Power Converter Control

Florian Dörfler, ETH Zurich

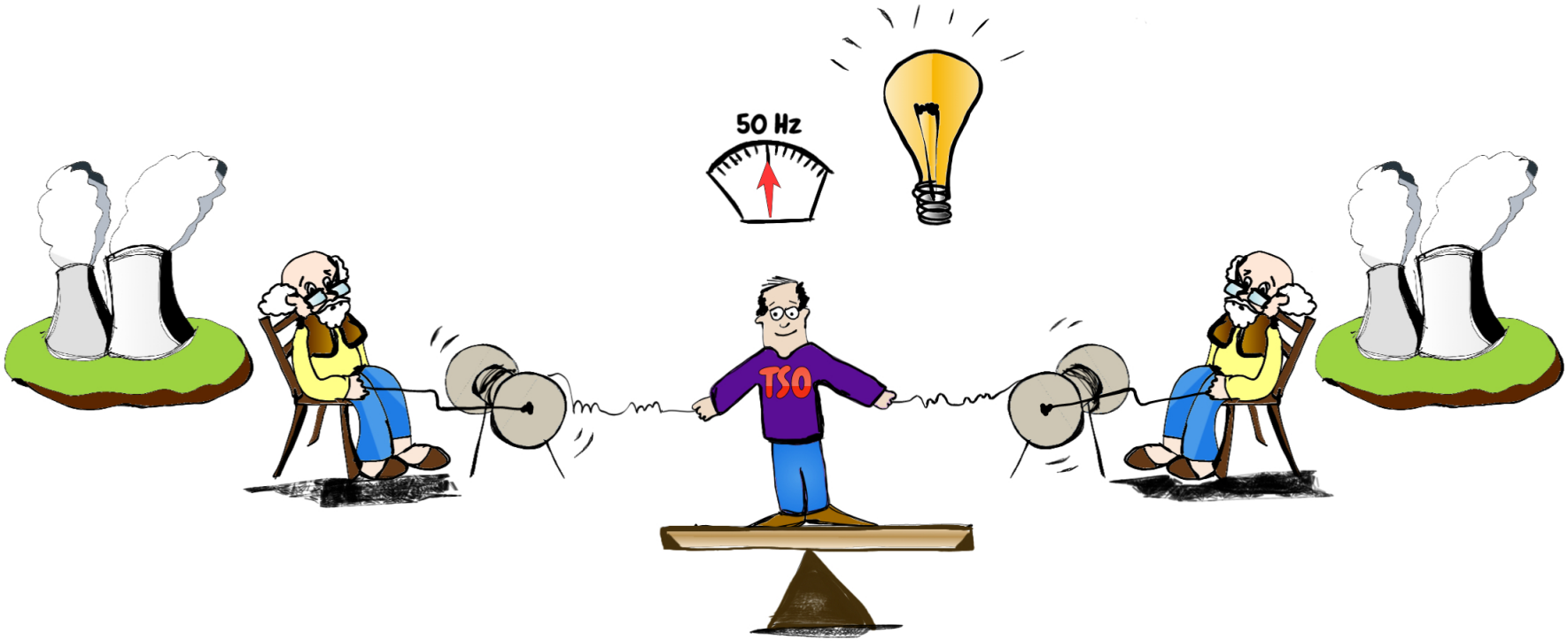
EECI School M19, June 20-23, 2022, Stockholm

Outline

- Virtual Synchronous Machine Emulation
- Droop Control
- Matching Control
- Virtual Oscillator Control (VOC)
- Time-Scale Separation Issues
- Comparison

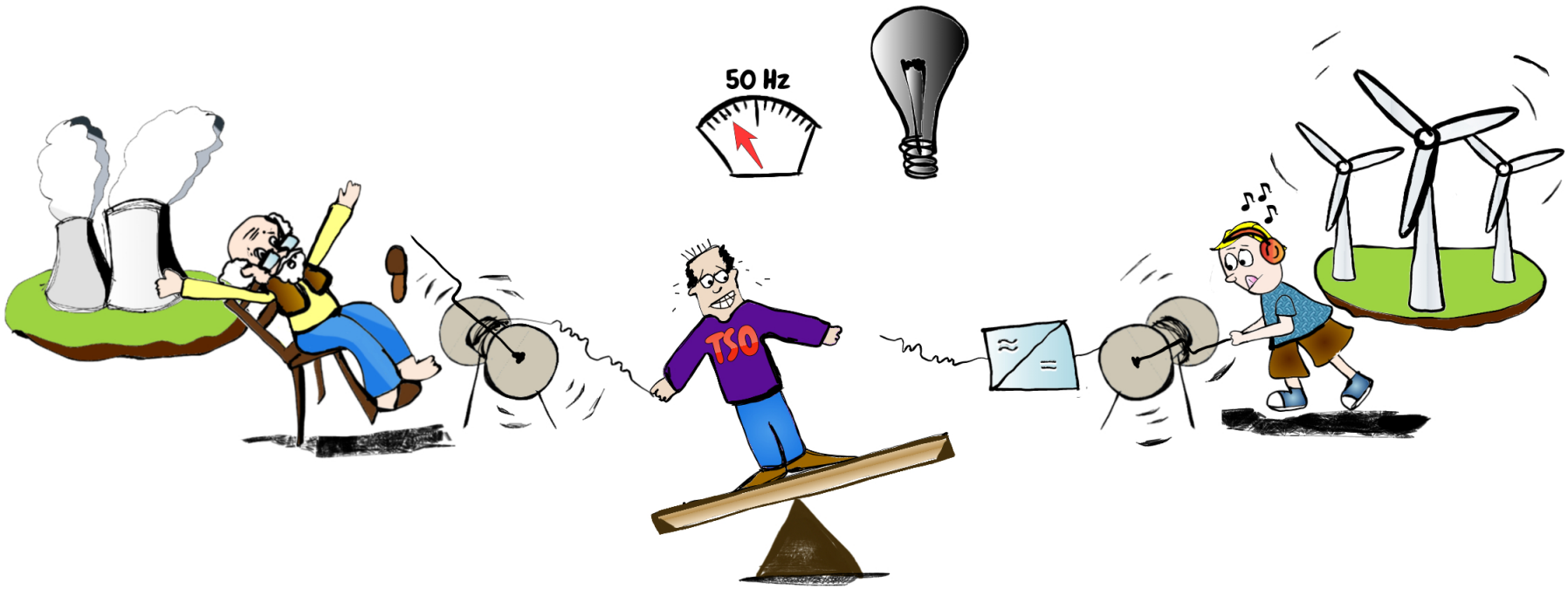
Cartoon summary of our approach

figure provided by Uroš Marković



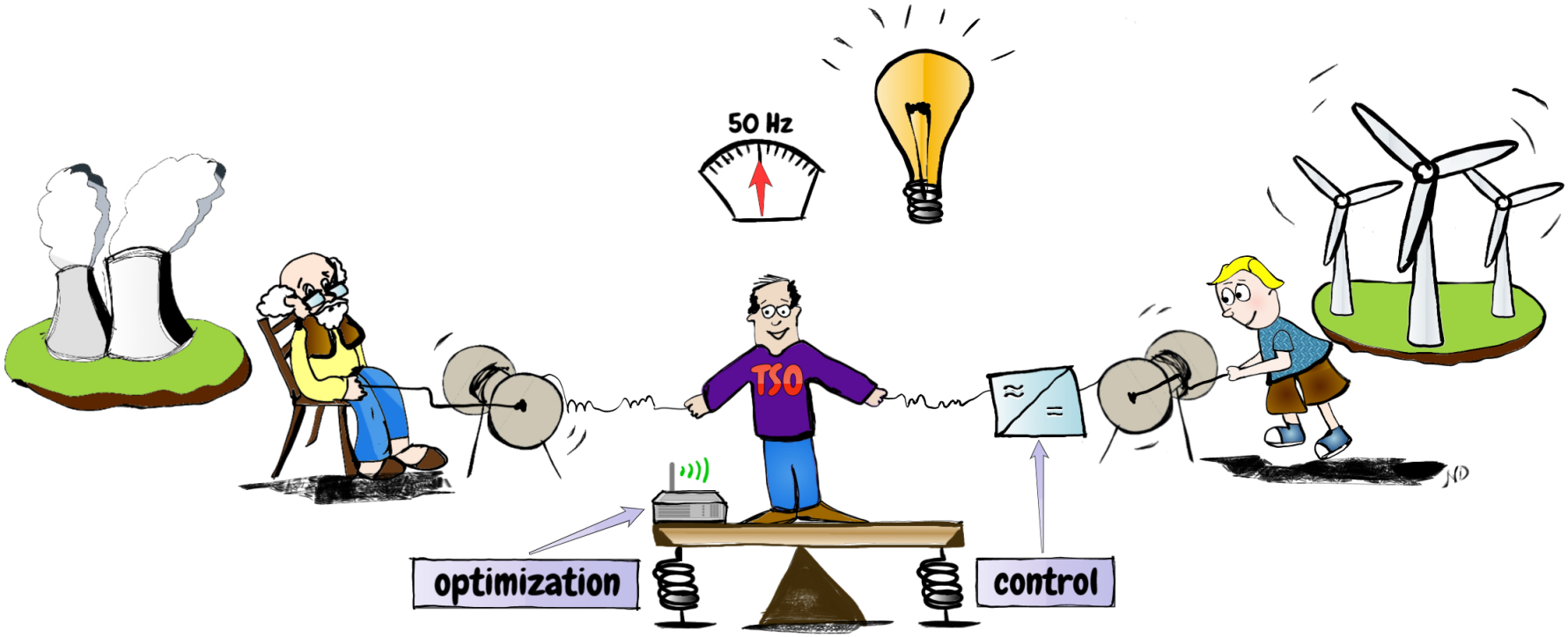
Cartoon summary of our approach

figure provided by Uroš Marković

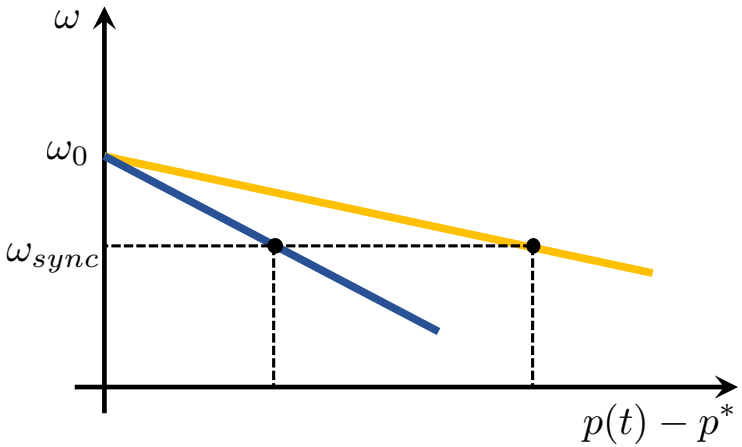


Cartoon summary of our approach

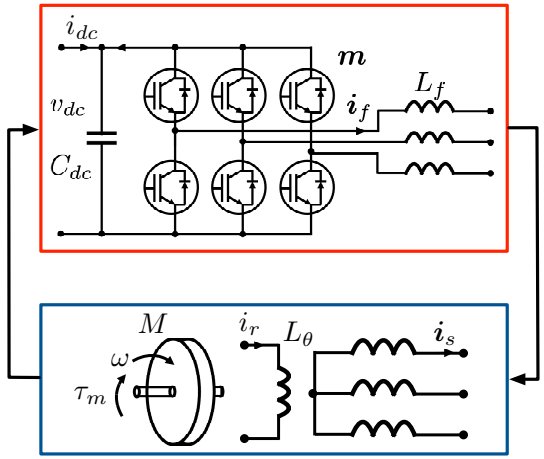
figure provided by Uroš Marković



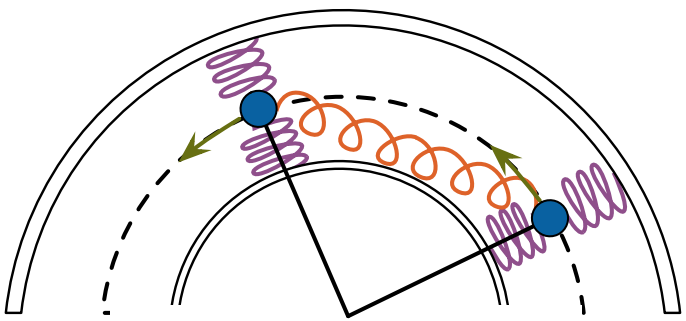
Overview of grid-forming control strategies



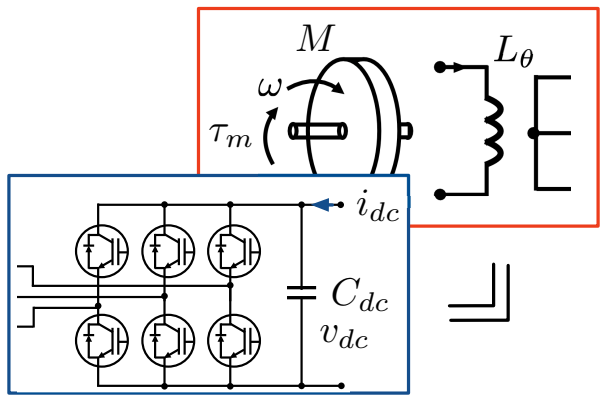
droop control



synchronous machine emulation



virtual oscillator control



matching control & duality

Outline

- Virtual Synchronous Machine Emulation
- Droop Control
- Matching Control
- Virtual Oscillator Control (VOC)
- Time-Scale Separation Issues
- Comparison

Naive baseline: virtual inertia emulation

IRELAND

Hybrid storage system looks to Ireland's services market

22 November 2016 by Sara Verbruggen , Be the first to comment

IRELAND: The pilot of a 576kW grid storage system using flywheels and batteries by Dublin-based Schwungrad Energie is lo… technology's deployment in Ireland's ancil…



Can Synthetic Inertia from Wind Power Stabilize Grids?

By Peter Fairley
Posted 7 Nov 2016 | 21:00 GMT

Pure-play battery or hybrid grid energy storage?

Oct 11, 2016 12:54 PM BST 0 Share



Improvement of Transient Response in Microgrids Using Virtual Inertia

Nimish Soni, *Student Member, IEEE*, Suryanarayana Doolla, *Member, IEEE*, and Mukul C. Chandorkar, *Member, IEEE*

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 28, NO. 2, MAY 2013 1373

Implementing Virtual Inertia in DFIG-Based Wind Power Generation

…mmadreza Fakhari Moghaddam Arani, *Student Member, IEEE*, and Ehab F. El-Saadany, *Senior Member, IEEE*

Virtual synchronous generators: A survey and new perspectives

Hassan Bevrani [a,b,*], Toshifumi Ise [b], Yushi Miura [b]

[a] *Dept. of Electrical and Computer Eng., University of Kurdistan, PO Box 416, Sanandaj, Iran*
[b] *Dept. of Electrical, Electronic and Information Eng., Osaka University, Osaka, Japan*

Dynamic Frequency Control Support: a Virtual Inertia Provided by Distributed Energy Storage to Isolated Power Systems

…Gauthier Delille, *Member, IEEE*, Bruno François, *Senior Member, IEEE*, and Gilles Malarange

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 28, NO. 2, MAY 2013

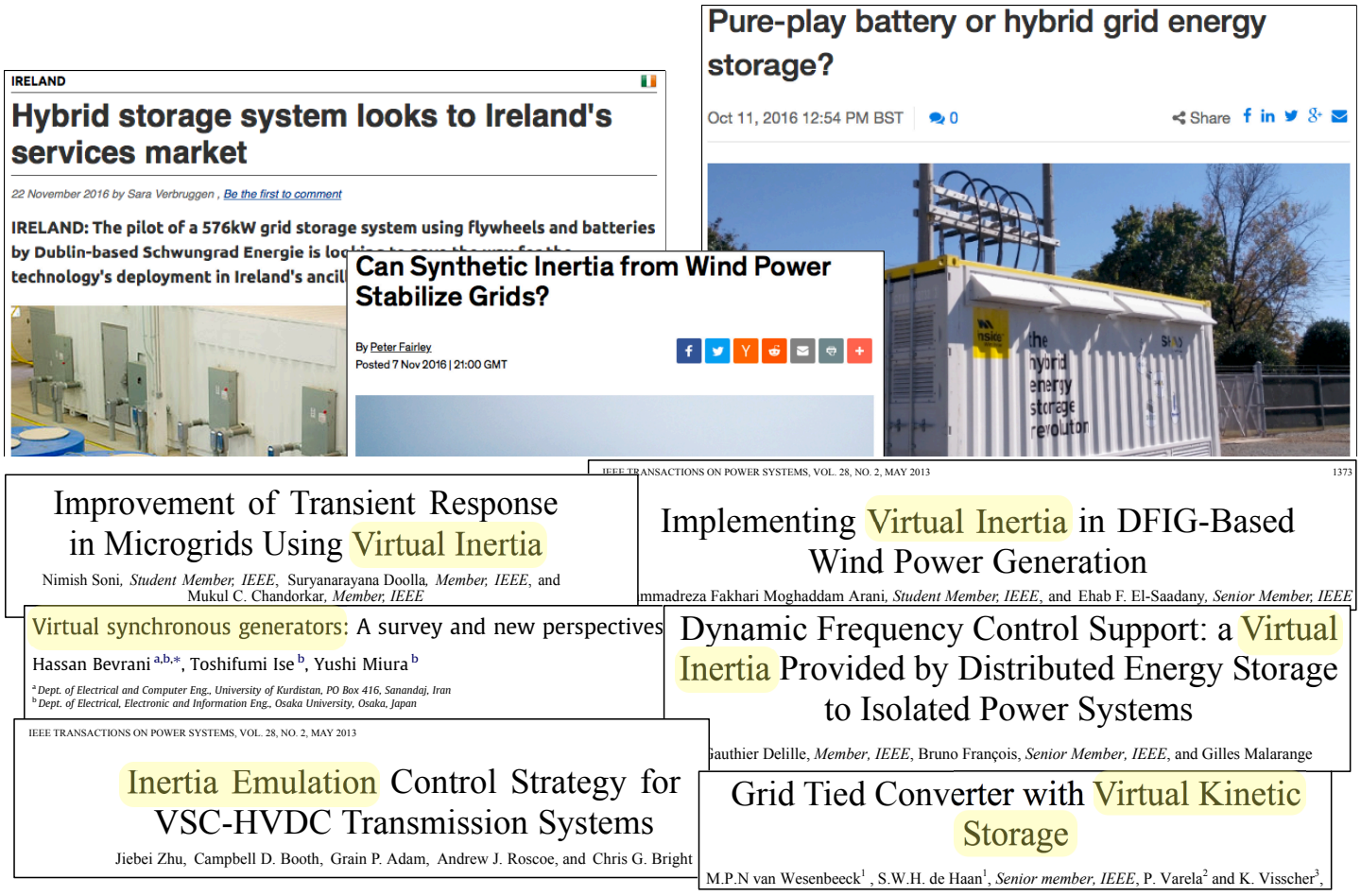
Inertia Emulation Control Strategy for VSC-HVDC Transmission Systems

Jiebei Zhu, Campbell D. Booth, Grain P. Adam, Andrew J. Roscoe, and Chris G. Bright

Grid Tied Converter with Virtual Kinetic Storage

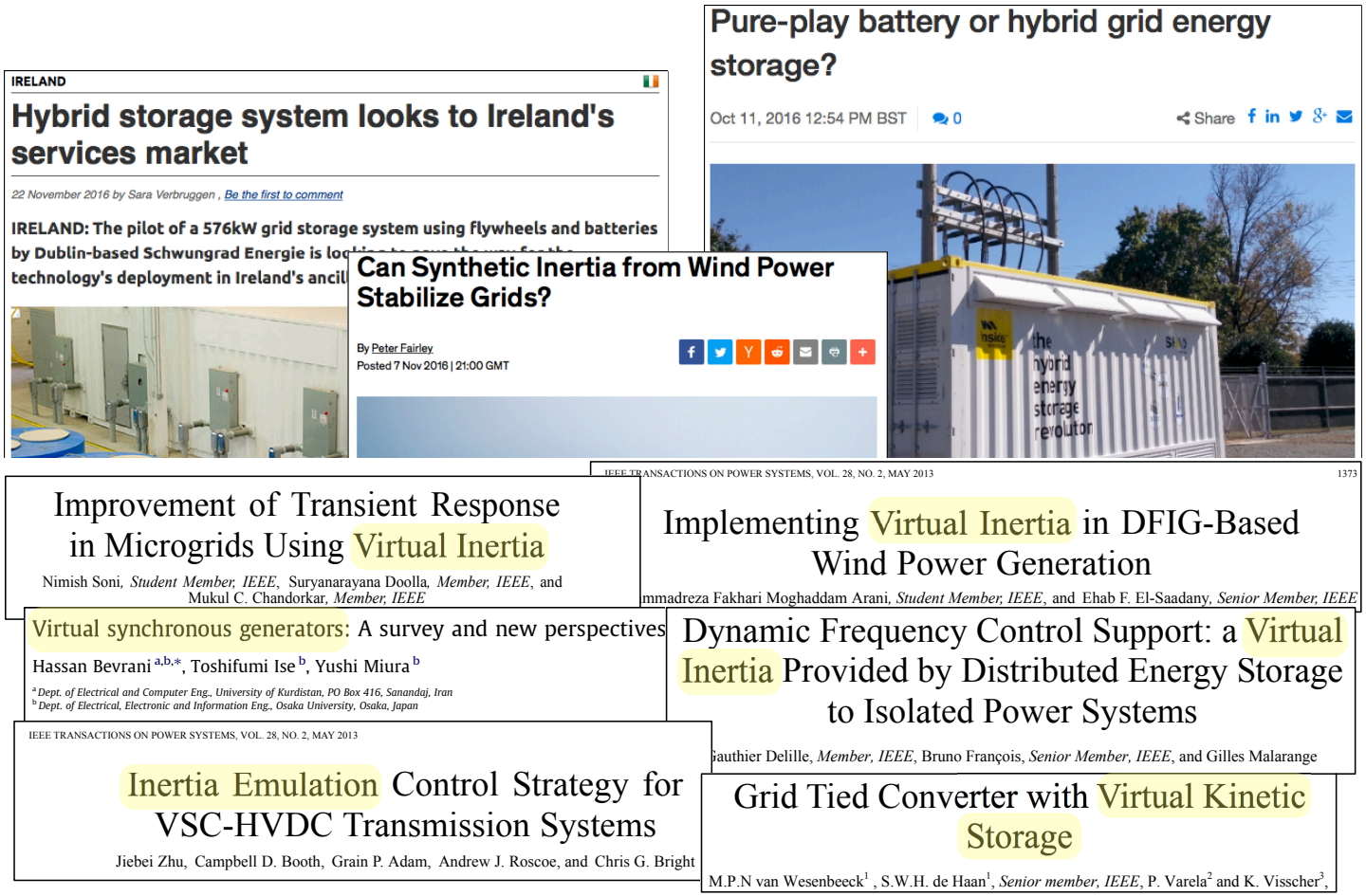
M.P.N van Wesenbeeck[1], S.W.H. de Haan[1], *Senior member, IEEE*, P. Varela[2] and K. Visscher[3],

Naive baseline: virtual inertia emulation



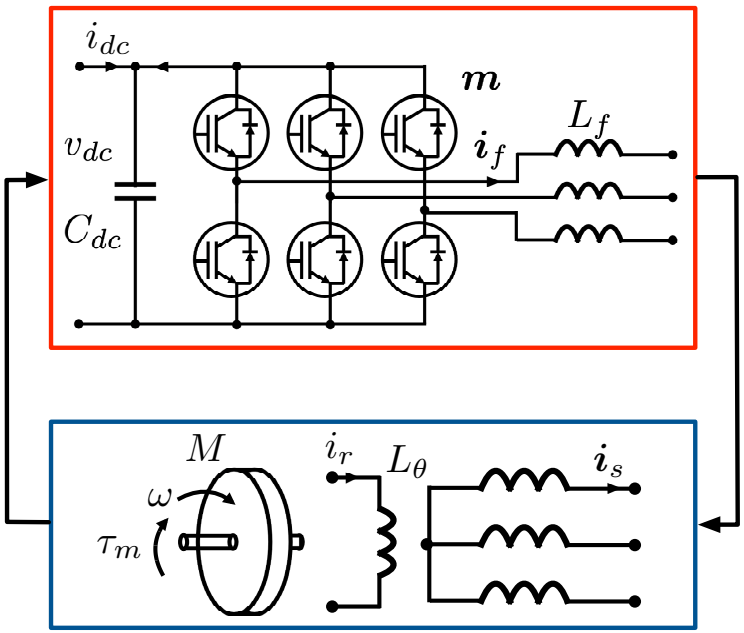
- **PD control** on $\omega(t)$: $\boldsymbol{M}\,\frac{d}{dt}\,\omega(t) + \boldsymbol{D}\,(\omega(t) - \omega_0) \;=\; P_{\text{generation}}(t) - P_{\text{demand}}(t)$

Naive baseline: virtual inertia emulation



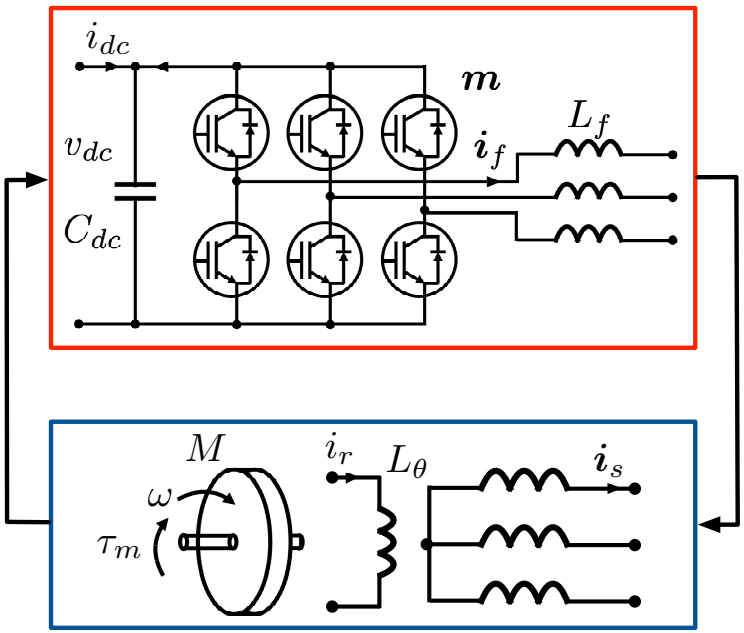
- **PD control** on $\omega(t)$: $\boldsymbol{M}\,\frac{d}{dt}\,\omega(t) + \boldsymbol{D}\,(\omega(t) - \omega_0) \;=\; P_{\text{generation}}(t) - P_{\text{demand}}(t)$
- there are **smarter implementations** at the cost of algorithmic complexity

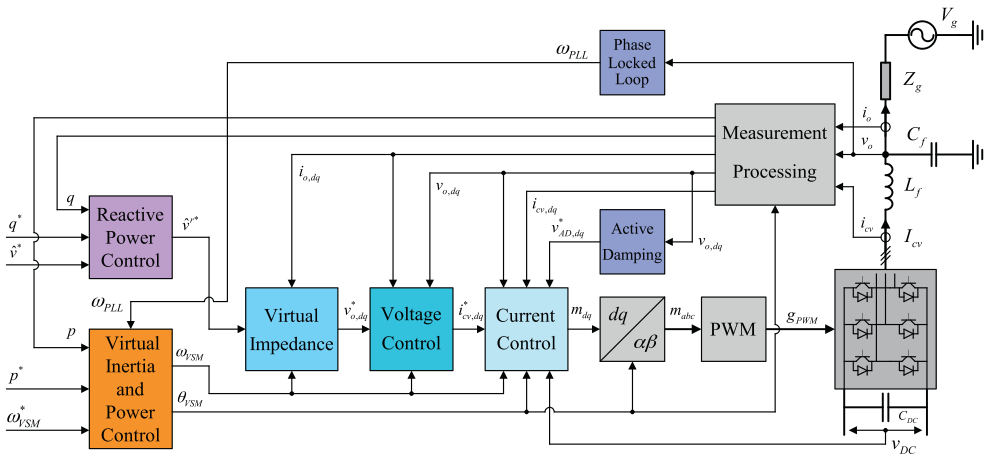
Virtual synchronous machine emulation



- **reference** : detailed model of synchronous generator + controls

Virtual synchronous machine emulation

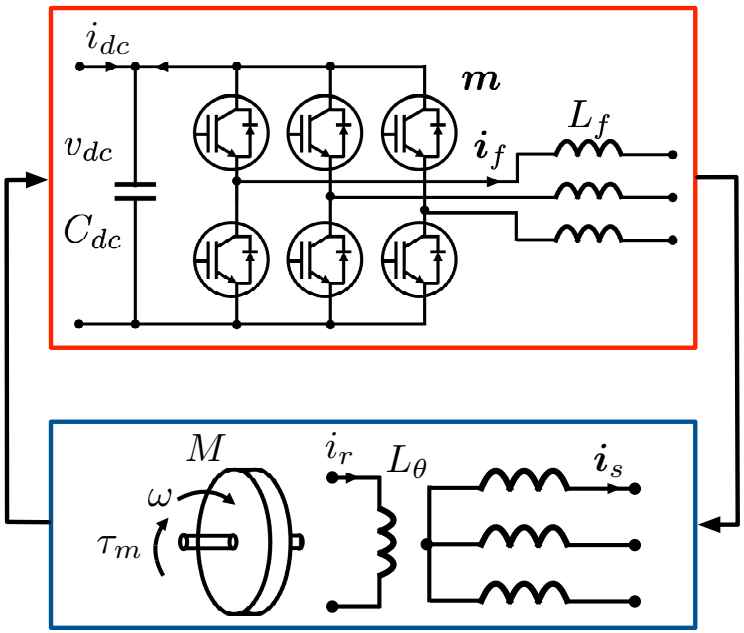


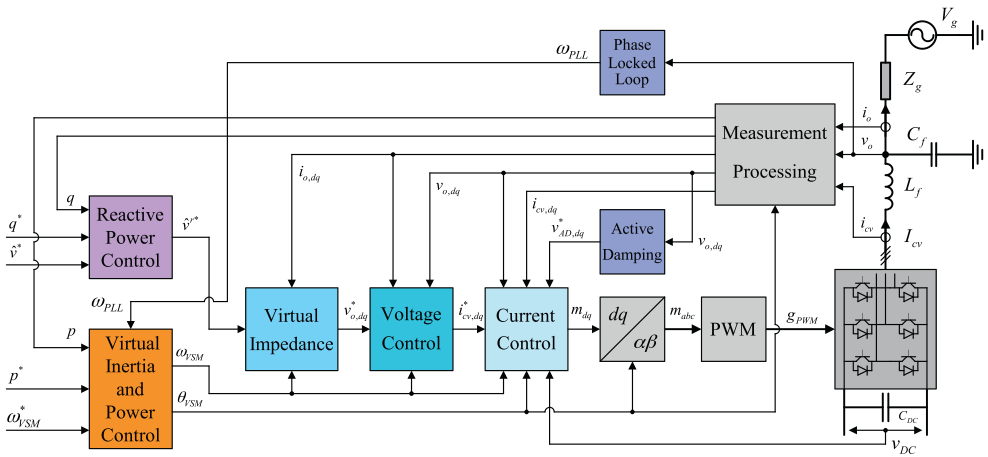


D'Arco et al., Electric Power Systems Research, 2015

- **reference** : detailed model of synchronous generator + controls
- **implementation** : low-pass filters for dissipation, virtual impedances for saturation, limiters,. . . tricks

Virtual synchronous machine emulation



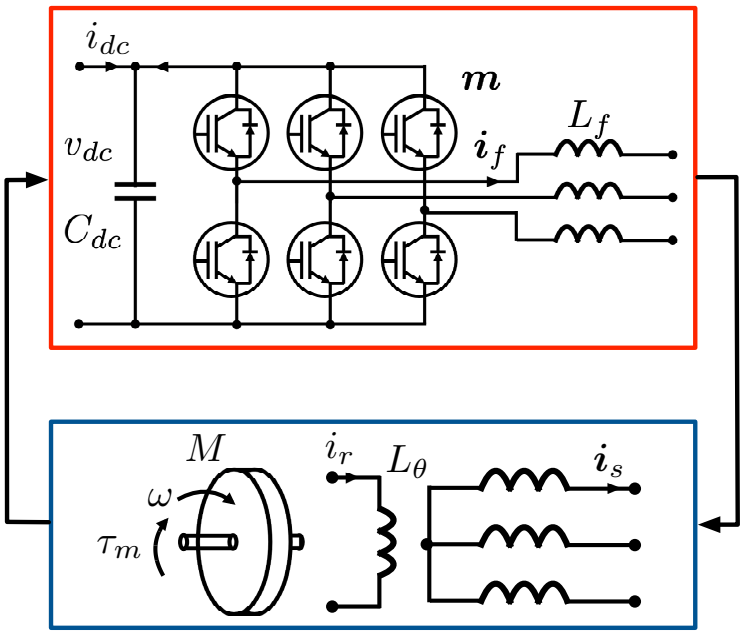


D'Arco et al., Electric Power Systems Research, 2015

- **reference** : detailed model of synchronous generator + controls
- **implementation** : low-pass filters for dissipation, virtual impedances for saturation, limiters,. . . tricks

→ most commonly **accepted solution** in **industry** (¿ backward compatibility ?)

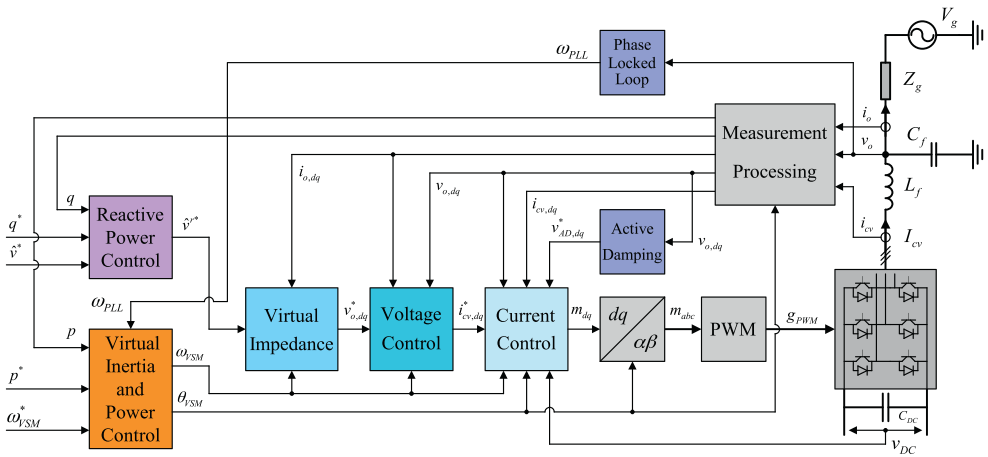
Virtual synchronous machine emulation



- **reference** : detailed model of synchronous generator + controls
- **implementation** : low-pass filters for dissipation, virtual impedances for saturation, limiters,. . . tricks

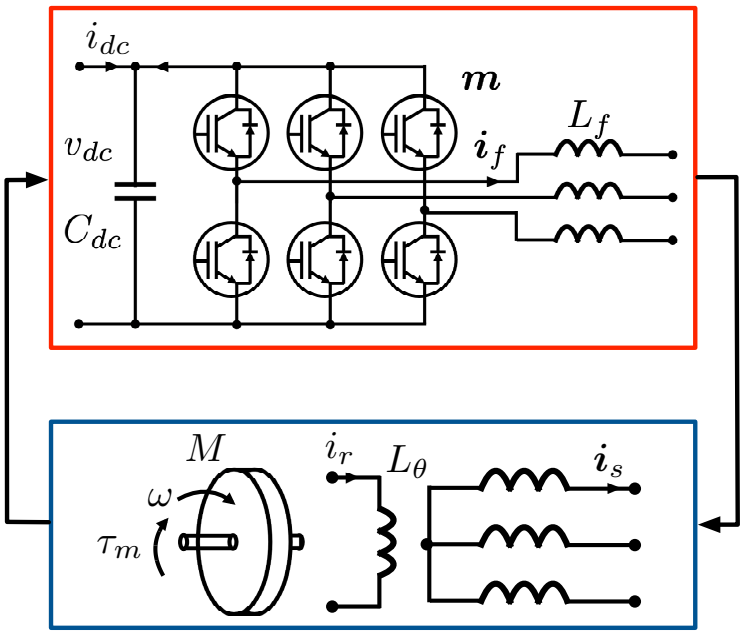
→ most commonly **accepted solution** in **industry** (¿ backward compatibility ?)

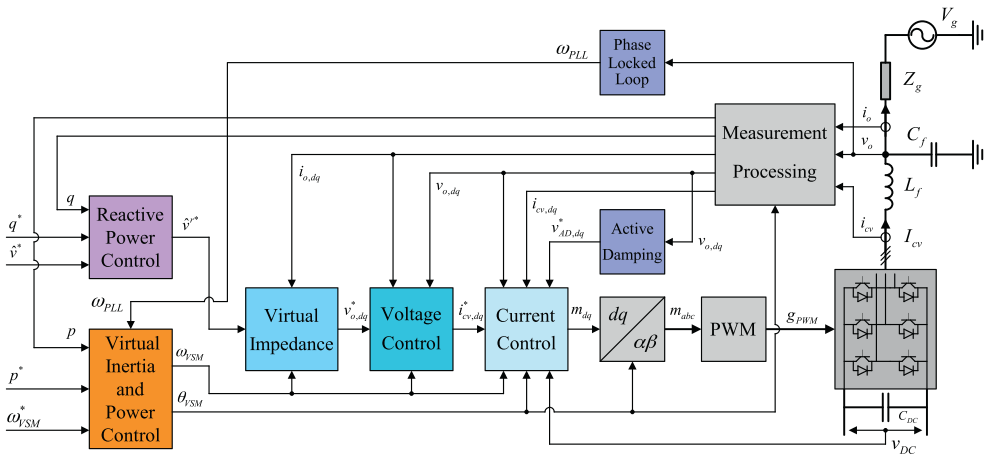
→ **over-parametrized** & ignores **DC source dynamics** and **limits**



D'Arco et al., Electric Power Systems Research, 2015

Virtual synchronous machine emulation





D'Arco et al., Electric Power Systems Research, 2015

- **reference** : detailed model of synchronous generator + controls
- **implementation** : low-pass filters for dissipation, virtual impedances for saturation, limiters,. . . tricks

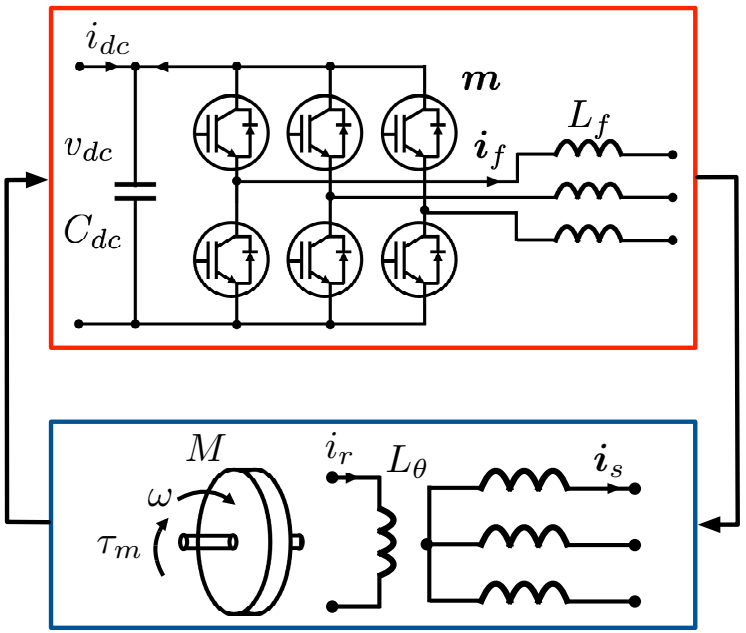
→ most commonly **accepted solution** in **industry** (¿ backward compatibility ?)

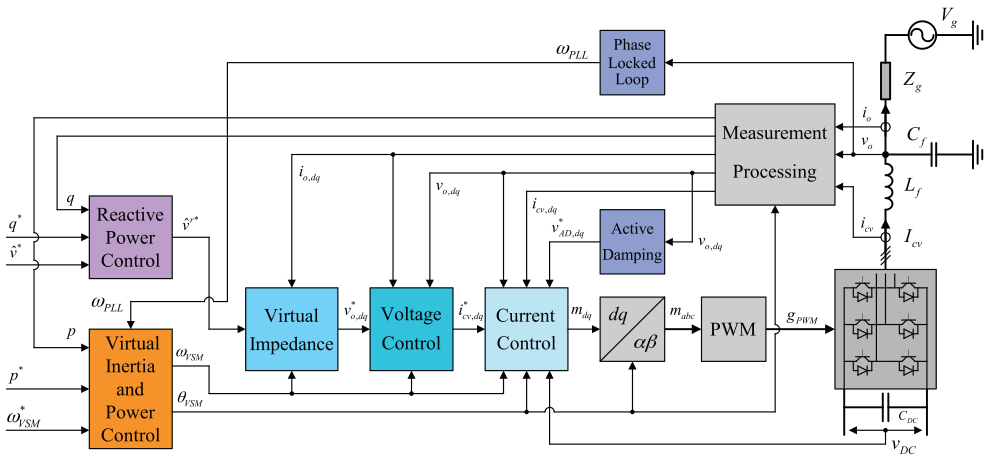
→ **over-parametrized** & ignores **DC source dynamics** and **limits**

→ **poor fit** for converter:

- converter: **fast** actuation & **no** significant **energy storage**
- machine: **slow** actuation & significant **energy storage**

Virtual synchronous machine emulation





D'Arco et al., Electric Power Systems Research, 2015

- **reference** : detailed model of synchronous generator + controls
- **implementation** : low-pass filters for dissipation, virtual impedances for saturation, limiters,. . . tricks

→ most commonly **accepted solution** in **industry** (¿ backward compatibility ?)

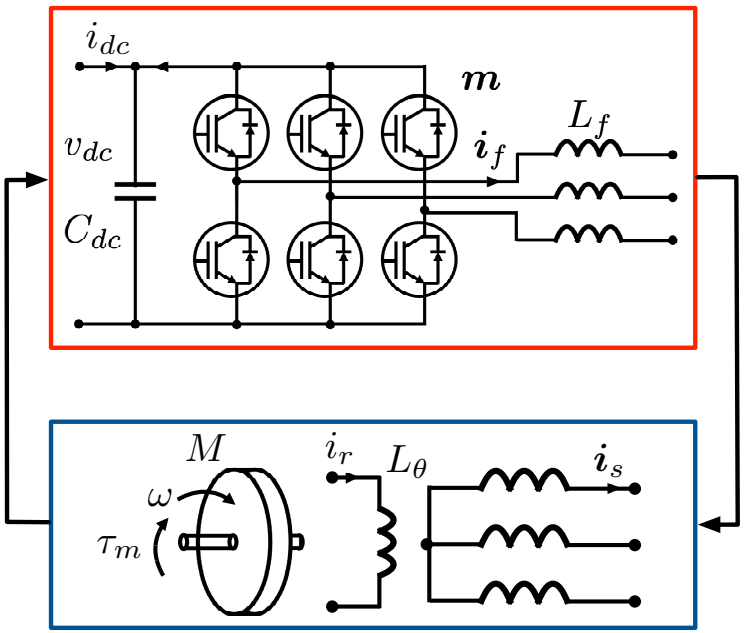
→ **over-parametrized** & ignores **DC source dynamics** and **limits**

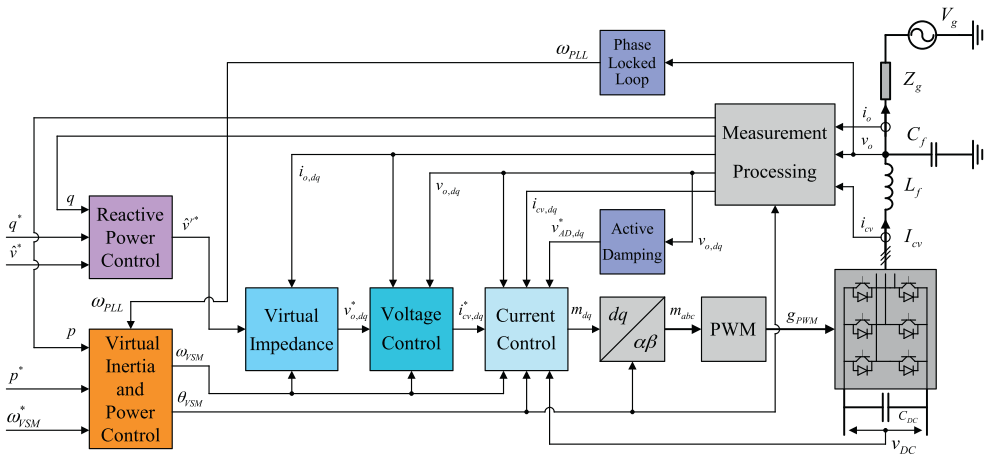
→ **poor fit** for converter:

  - converter: **fast** actuation & **no** significant **energy storage**
  - machine: **slow** actuation & significant **energy storage**

→ **performs poorly** post-fault

Virtual synchronous machine emulation





D'Arco et al., Electric Power Systems Research, 2015

- **reference** : detailed model of synchronous generator + controls
- **implementation** : low-pass filters for dissipation, virtual impedances for saturation, limiters,. . . tricks

→ most commonly **accepted solution** in **industry** (¿ backward compatibility ?)

→ **over-parametrized** & ignores **DC source dynamics** and **limits**

→ **poor fit** for converter:

  - converter: **fast** actuation & **no** significant **energy storage**
  - machine: **slow** actuation & significant **energy storage**

→ **performs poorly** post-fault

→ **analysis** is **hopeless** (hard at best)

Details & initial conditions for further reading

Virtual synchronous generators: A survey and new perspectives

Hassan Bevrani [a,b,*], Toshifumi Ise [b], Yushi Miura [b]

[a] Dept. of Electrical and Computer Eng., University of Kurdistan, PO Box 416, Sanandaj, Iran
[b] Dept. of Electrical, Electronic and Information Eng., Osaka University, Osaka, Japan

ARTICLE INFO

Article history:
Received 31 December 2012
Received in revised form 12 June 2013
Accepted 13 July 2013

Keywords:
Virtual inertia
Renewable energy
VSG
Frequency control
Voltage control
Microgrid

ABSTRACT

In comparison of the conventional bulk power plants, in which the synchronous machines dominate, the distributed generator (DG) units have either very small or no rotating mass and damping property. With growing the penetration level of DGs, the impact of low inertia and damping effect on the grid stability and dynamic performance increases. A solution towards stability improvement of such a grid is to provide virtual inertia by virtual synchronous generators (VSGs) that can be established by using short term energy storage together with a power inverter and a proper control mechanism.

The present paper reviews the fundamentals and main concept of VSGs, and their role to support the power grid control. Then, a VSG-based frequency control scheme is addressed, and the paper is focused on the poetical role of VSGs in the grid frequency regulation task. The most important VSG topologies with a survey on the recent works/achievements are presented. Finally, the relevant key issues, main technical challenges, further research needs and new perspectives are emphasized.

© 2013 Elsevier Ltd. All rights reserved.

Comprehensive assessment of virtual synchronous machine based voltage source converter controllers

ISSN 1751-8687
Received on 7th September 2016
Revised 28th December 2016
Accepted on 15th February 2017
E-First on 24th April 2017
doi: 10.1049/iet-gtd.2016.1423
www.ietdl.org

Hasan Alrajhi Alsiraji[1,2] ✉, *Ramadan El-Shatshat*[1]

[1]*Department of Electrical and Computer Engineering, University of Waterloo, 200 University Ave W, Waterloo, ON, Canada*
[2]*Department of Electrical Engineering, Umm al-Qura University, Al-Taif Road, Mecca 24382, Saudi Arabia*
✉ *E-mail: halsiraj@uwaterloo.ca*

Abstract: The substantial potential for the integration of renewable energy into power systems using power electronics converters might result in stability issues because of a lack of inertia. For this reason, this study introduces the concept of a virtual synchronous machine (VSM) control algorithm that emulates the properties of traditional synchronous machines. The literature includes references to several differently structured control algorithms. However, synchronous machine inertia and damping characteristics must be mimicked, which makes the cost and simplicity of implementation important from an economic perspective. This study presents a comprehensive comparison of VSM control algorithms. The most significant factor investigated in the work presented in this study is the viability of VSM algorithms during the kind of abnormal operation that might raise instability issues with respect to practical discrete time operation. The test system used in this study, which was simulated in a PSCAD/EMTDC environment, consisted of simulated voltage source converters based on a fully detailed switching model with two AC voltage levels. The results indicate a significant outcome that can facilitate a determination of the most effective VSM control algorithm.

Synchronverters: Inverters That Mimic Synchronous Generators

Qing-Chang Zhong, *Senior Member, IEEE*, and George Weiss

***Abstract*—In this paper, the idea of operating an inverter to mimic a synchronous generator (SG) is motivated and developed. We call the inverters that are operated in this way synchronverters. Using synchronverters, the well-established theory/algorithms used to control SGs can still be used in power systems where a significant proportion of the generating capacity is inverter-based. We describe the dynamics, implementation, and operation of synchronverters. The real and reactive power delivered by synchronverters connected in parallel and operated as generators can be automatically shared using the well-known frequency- and voltage-drooping mechanisms. Synchronverters can be easily operated also in island mode, and hence, they provide an ideal solution for microgrids or smart grids. Both simulation and experimental results are given to verify the idea.**

called inverters, to interface with the public-utility grid. For example, wind turbines are most effective if free to generate at variable frequency, and so, they require conversion from variable frequency ac to dc to ac; small gas-turbines with direct-drive generators operate at high frequency and also require ac to dc to ac conversion; photovoltaic arrays require dc–ac conversion. This means that more and more inverters will be connected to the grid and will eventually dominate power generation.

The current paradigm in the control of wind- or solar-power generators is to extract the maximum power from the power source and inject them all into the power grid (see, for example,

Control System Tuning and Stability Analysis of Virtual Synchronous Machines

Salvatore D'Arco* Jon Are Suul*† Olav B. Fosso†

*SINTEF Energy Research
7465 Trondheim, Norway
salvatore.darco@sintef.no

†Department of Electric Power Engineering
Norwegian University of Science and Technology
7495 Trondheim, Norway

***Abstract*—Virtual Synchronous Machines (VSMs) have been introduced as a control concept for emulating the behavior of traditional synchronous machines with power electronic converters. This paper analyses a VSM control scheme where an outer loop for inertia emulation provides references for two cascaded voltage**

as well as stand-alone mode, and can ensure load sharing in the same way as the droop-based controls for UPS systems and microgrids [9]-[12]. Moreover, a VSM inherently includes the function of inertia emulation, which is expected to be important in future power systems with limited physical inertia [4], [6].

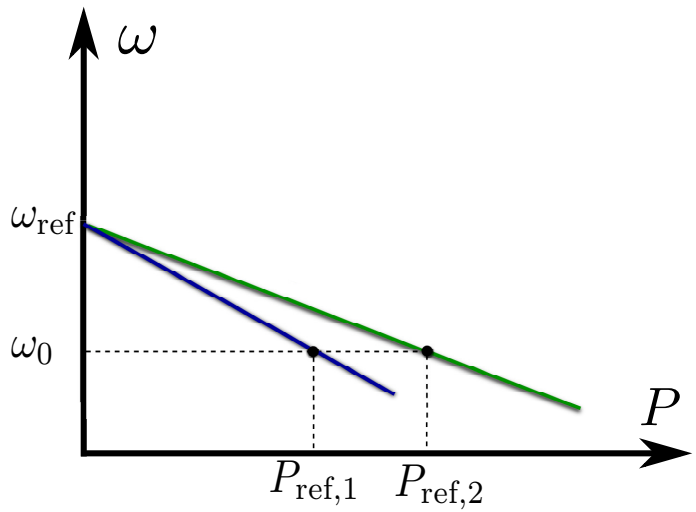
Outline

- Virtual Synchronous Machine Emulation
- Droop Control
- Matching Control
- Virtual Oscillator Control (VOC)
- Time-Scale Separation Issues
- Comparison

Droop as simplest reference model

- **frequency control** by mimicking $P - \omega$ droop property of synchronous machine:

$$D\left(\omega - \omega_{\text{ref}}\right) \;=\; P_{\text{ref}} - P$$



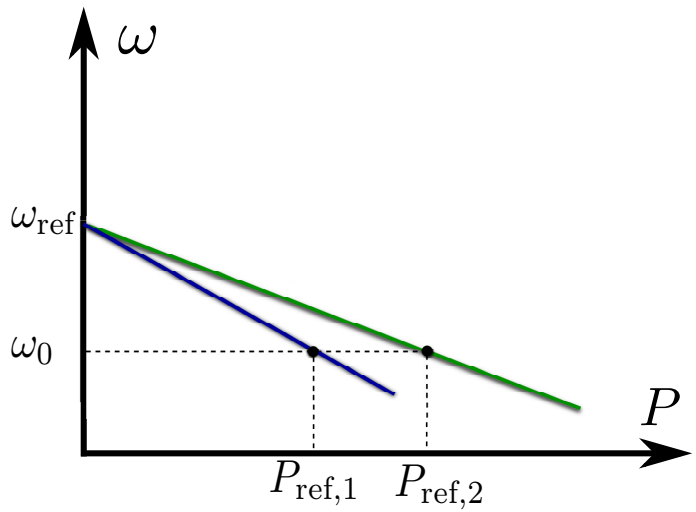
Droop as simplest reference model

- **frequency control** by mimicking $P - \omega$ droop property of synchronous machine:

$$D\left(\omega - \omega_{\text{ref}}\right) \;=\; P_{\text{ref}} - P$$

- **voltage control** via $Q - \|\boldsymbol{v}\|$ droop:

$$\tfrac{d}{dt}\|v\| = -c_1(\|\boldsymbol{v}\| - v_{\text{ref}}) - c_2(Q - Q_{\text{ref}})$$



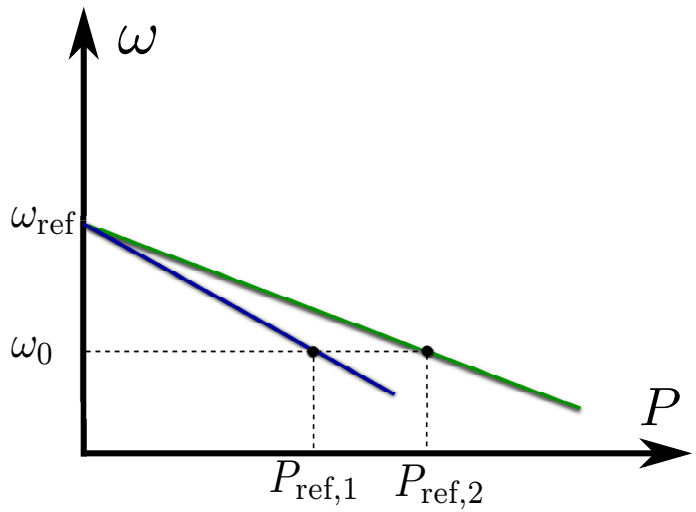
Droop as simplest reference model

- **frequency control** by mimicking $P - \omega$ droop property of synchronous machine:

$$D\,(\omega - \omega_{\text{ref}}) \;=\; P_{\text{ref}} - P$$

- **voltage control** via $Q - \|\boldsymbol{v}\|$ droop:

$$\tfrac{d}{dt}\|v\| = -c_1(\|\boldsymbol{v}\| - v_{\text{ref}}) - c_2(Q - Q_{\text{ref}})$$



→ **reference** are generator controls

→ **direct control** of (P, ω) and $(Q, \|\boldsymbol{v}\|)$ **assuming they are independent** (approx. true only near steady state)

→ **two SISO loops** to control a **nonlinear MIMO system**

→ ignores DC **source dynamics**

→ requires **tricks in implementation** similar to virtual synchronous machine

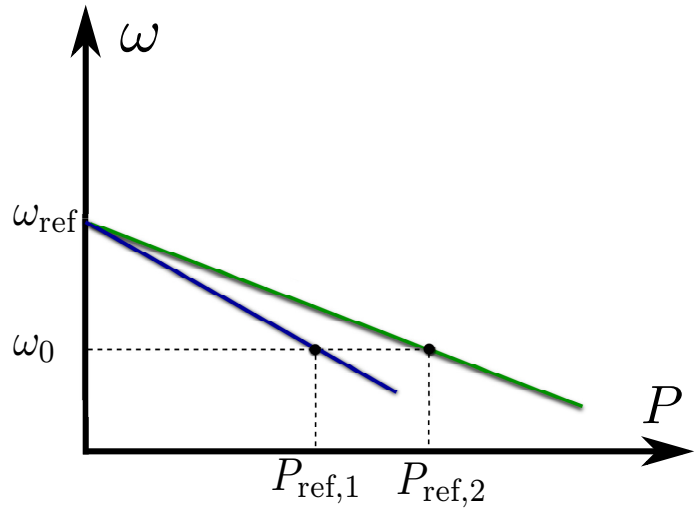
Droop as simplest reference model

- **frequency control** by mimicking $P - \omega$ droop property of synchronous machine:

$$D\left(\omega - \omega_{\text{ref}}\right) \;=\; P_{\text{ref}} - P$$

- **voltage control** via $Q - \|\boldsymbol{v}\|$ droop:

$$\frac{d}{dt}\|v\| = -c_1(\|\boldsymbol{v}\| - v_{\text{ref}}) - c_2(Q - Q_{\text{ref}})$$



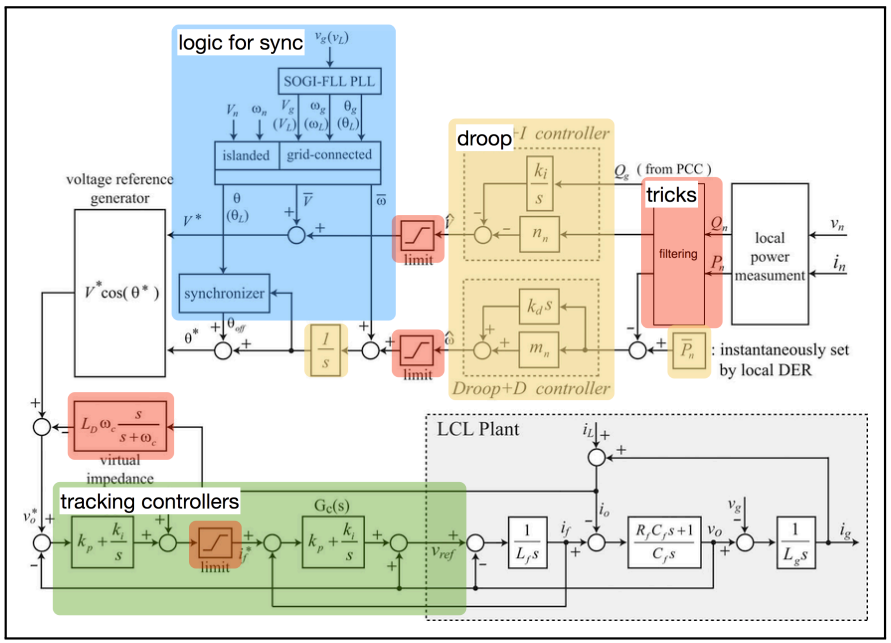
→ **reference** are generator controls

→ **direct control** of (P, ω) and $(Q, \|\boldsymbol{v}\|)$ **assuming they are independent** (approx. true only near steady state)

→ **two SISO loops** to control a **nonlinear MIMO system**

→ ignores DC **source dynamics**

→ requires **tricks in implementation** similar to virtual synchronous machine



Derivation of ideal droop angle dynamics

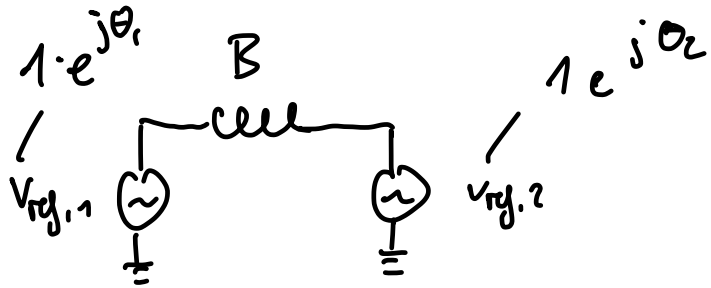
(covered on the board)

Assume:
- no state / power / energy constraints
- AC tracking controls work perfectly
  - ⇝ terminal converter voltage = reference voltage
- neglect voltage control (assume all voltages $|v| = 1$)
- lossless network

reduced problem scenario: (two converters)

droop control: $D_i(\omega_i - \omega_{ref}) = P_{i,ref} - P_{i,inj}$

(D_i: droop)



define: $\theta_i = \int \omega_i - \omega_{ref}\, dt$

⇝
$$D_1 \dot{\theta}_1 = P_{1,ref} - B \sin(\theta_1 - \theta_2)$$
$$D_2 \dot{\theta}_2 = P_{2,ref} - B \sin(\theta_2 - \theta_1)$$

= coupled oscillator dynamics (synchronize)

Droop control is *de facto* baseline solution

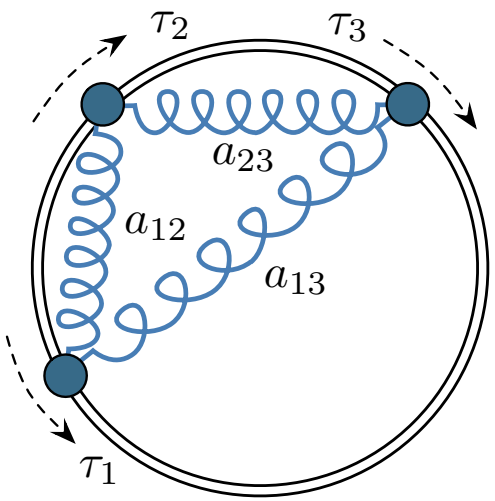
- after (lots of) deliberate tuning, it **works well locally** near steady state
- admits both **grid-forming & grid-following** implementations
- simplified droop models are amenable to theoretic **analysis & certificates**

Droop control is *de facto* baseline solution

- after (lots of) deliberate tuning, it **works well locally** near steady state
- admits both **grid-forming & grid-following** implementations
- simplified droop models are amenable to theoretic **analysis & certificates**

(simplified) **frequency droop** $=$ coupled oscillators

$$\underbrace{\frac{d}{dt}\theta_i}_{\frac{d}{dt}\text{ terminal voltage}} = \underbrace{\tau_i}_{\text{active power set-point}} \underbrace{- \sum_j a_{ij} \sin(\theta_i - \theta_j)}_{\text{active power flows from grid}}$$

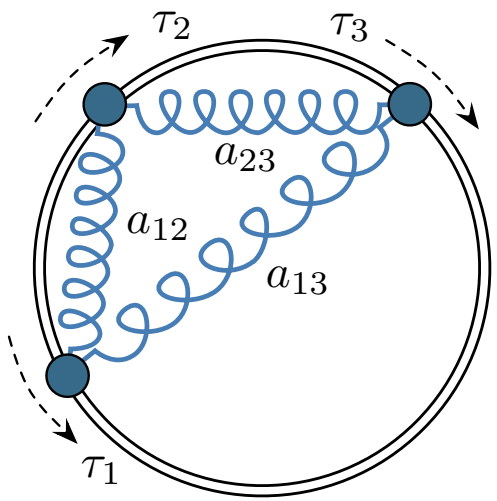


Droop control is *de facto* baseline solution

- after (lots of) deliberate tuning, it **works well locally** near steady state
- admits both **grid-forming & grid-following** implementations
- simplified droop models are amenable to theoretic **analysis & certificates**

(simplified) **frequency droop** = coupled oscillators

$$\underbrace{\frac{d}{dt}\theta_i}_{\frac{d}{dt}\text{ terminal voltage}} = \underbrace{\tau_i}_{\text{active power set-point}} \underbrace{- \sum_j a_{ij} \sin(\theta_i - \theta_j)}_{\text{active power flows from grid}}$$



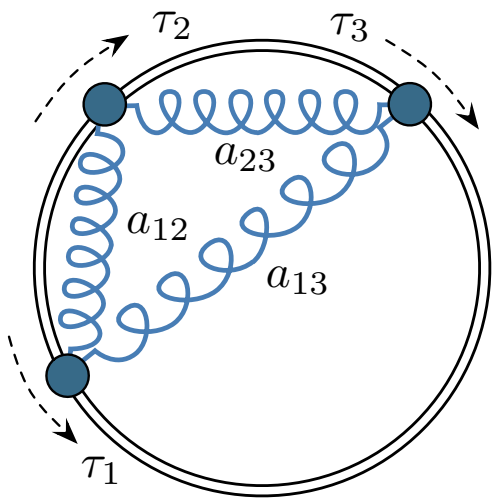
$\Rightarrow$ part of many **grid codes & ancillary service** markets

Droop control is *de facto* baseline solution

- after (lots of) deliberate tuning, it **works well locally** near steady state
- admits both **grid-forming & grid-following** implementations
- simplified droop models are amenable to theoretic **analysis & certificates**

(simplified) **frequency droop** = coupled oscillators

$$\underbrace{\frac{d}{dt}\theta_i}_{\frac{d}{dt}\text{ terminal voltage}} = \underbrace{\tau_i}_{\text{active power set-point}} \underbrace{- \sum_j a_{ij} \sin(\theta_i - \theta_j)}_{\text{active power flows from grid}}$$



$\Rightarrow$ part of many **grid codes & ancillary service** markets

ø **poor post-fault performance** due to delays, wind-up, decoupling, SISO, …

ø **no stability certificates** for detailed, nonlinear, & interconnected systems

ø unclear if droop control is the **long-term solution (?)** for low-inertia systems

Details & initial condition for further reading

Conditions for stability of droop-controlled inverter-based microgrids☆

Johannes Schiffer [a,1], Romeo Ortega [b], Alessandro Astolfi [c,d], Jörg Raisch [a,e], Tevfik Sezi [f]

[a] Technische Universität Berlin, Einsteinufer 11, 10587 Berlin, Germany
[b] Laboratoire des Signaux et Systémes, École Supérieure d' Electricité (SUPELEC), Gif-sur-Yvette 91192, France
[c] Department of Electrical and Electronic Engineering, Imperial College London, London SW7 2AZ, UK
[d] Dipartimento di Ing. Civile e Ing. Informatica, University of Rome, Tor Vergata, 00133 Rome, Italy
[e] Max-Planck-Institut für Dynamik komplexer technischer Systeme, Sandtorstr. 1, 39106 Magdeburg, Germany
[f] Siemens AG, Smart Grid Division, Energy Automation, Humboldtstr. 59, 90459 Nuremberg, Germany

ARTICLE INFO

Article history:
Received 8 May 2013
Received in revised form
21 November 2013
Accepted 19 May 2014
Available online 2 September 2014

Keywords:
Microgrid control
Microgrid stability
Smart grid applications
Inverters

ABSTRACT

We consider the problem of stability analysis for droop-controlled inverter-based microgrids with meshed topologies. The inverter models include variable frequencies as well as voltage amplitudes. Conditions on the tuning gains and setpoints for frequency and voltage stability, together with desired active power sharing, are derived in the paper. First, we prove that for all practical choices of these parameters global boundedness of trajectories is ensured. Subsequently, assuming the microgrid is lossless, a port-Hamiltonian description is derived, from which sufficient conditions for stability are given. Finally, we propose for generic lossy microgrids a design criterion for the controller gains and setpoints such that a desired steady-state active power distribution is achieved. The analysis is validated via simulation on a microgrid based on the CIGRE (Conseil International des Grands Réseaux Electriques) benchmark medium voltage distribution network.

© 2014 Elsevier Ltd. All rights reserved.

Synchronization and power sharing for droop-controlled inverters in islanded microgrids☆

John W. Simpson-Porco [1], Florian Dörfler, Francesco Bullo

Center for Control, Dynamical Systems, and Computation, University of California, Santa Barbara, CA 93106, USA

ARTICLE INFO

Article history:
Received 8 November 2012
Received in revised form
20 February 2013
Accepted 2 May 2013
Available online 14 June 2013

Keywords:
Inverters
Power-system control
Smart power applications
Synchronization
Coupled oscillators
Kuramoto model
Distributed control

ABSTRACT

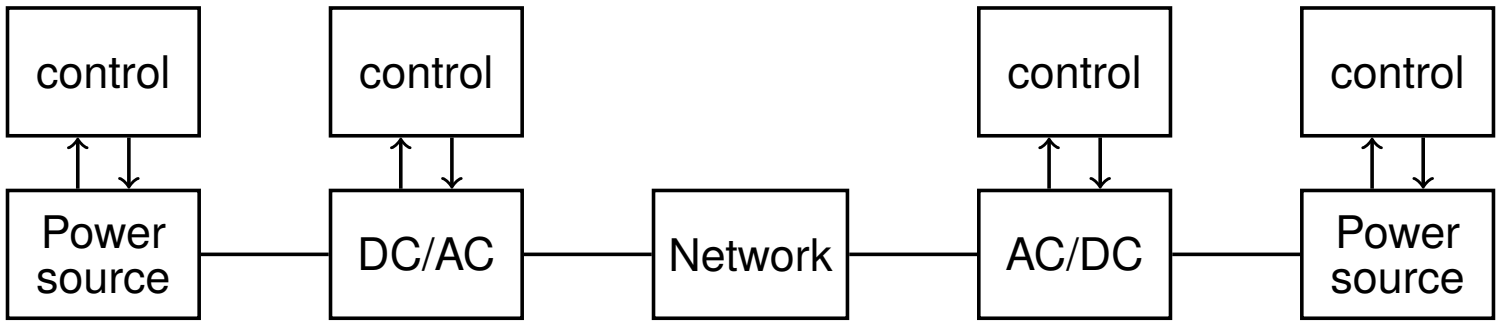
Motivated by the recent and growing interest in smart grid technology, we study the operation of DC/AC inverters in an inductive microgrid. We show that a network of loads and DC/AC inverters equipped with power-frequency droop controllers can be cast as a Kuramoto model of phase-coupled oscillators. This novel description, together with results from the theory of coupled oscillators, allows us to characterize the behavior of the network of inverters and loads. Specifically, we provide a necessary and sufficient condition for the existence of a synchronized solution that is unique and locally exponentially stable. We present a selection of controller gains leading to a desirable sharing of power among the inverters, and specify the set of loads which can be serviced without violating given actuation constraints. Moreover, we propose a distributed integral controller based on averaging algorithms, which dynamically regulates the system frequency in the presence of a time-varying load. Remarkably, this distributed-averaging integral controller has the additional property that it preserves the power sharing properties of the primary droop controller. Our results hold for any acyclic network topology, and hold without assumptions on identical line admittances or voltage magnitudes.

© 2013 Elsevier Ltd. All rights reserved.

Outline

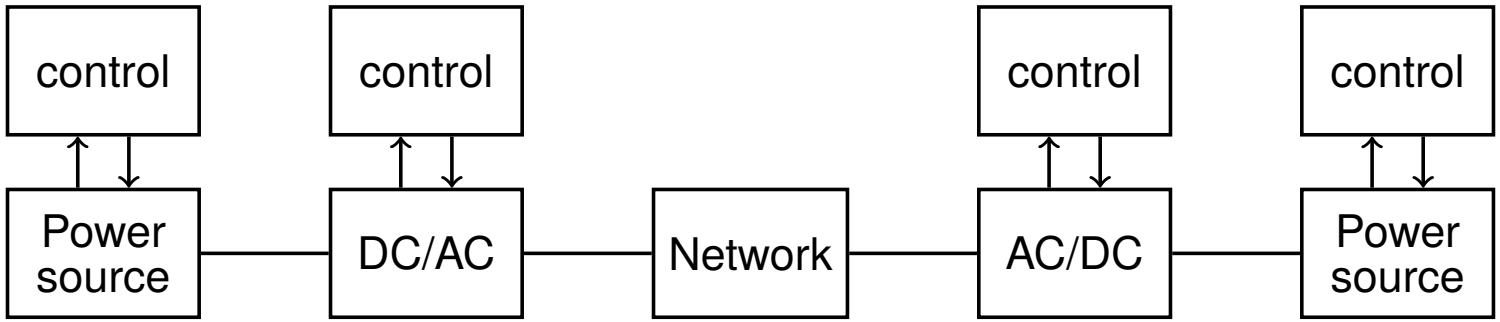
- Virtual Synchronous Machine Emulation
- Droop Control
- Matching Control
- Virtual Oscillator Control (VOC)
- Time-Scale Separation Issues
- Comparison

Power sources & signal transformers

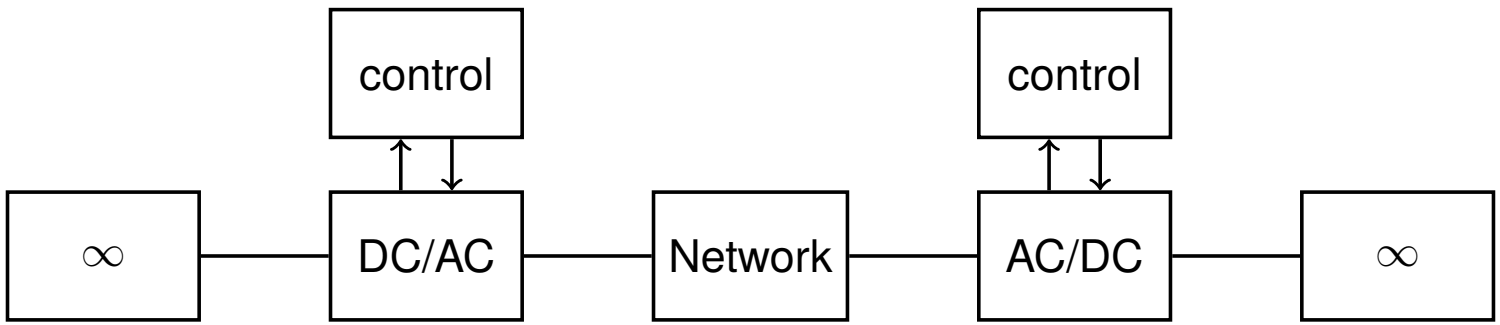


complete system model

Power sources & signal transformers

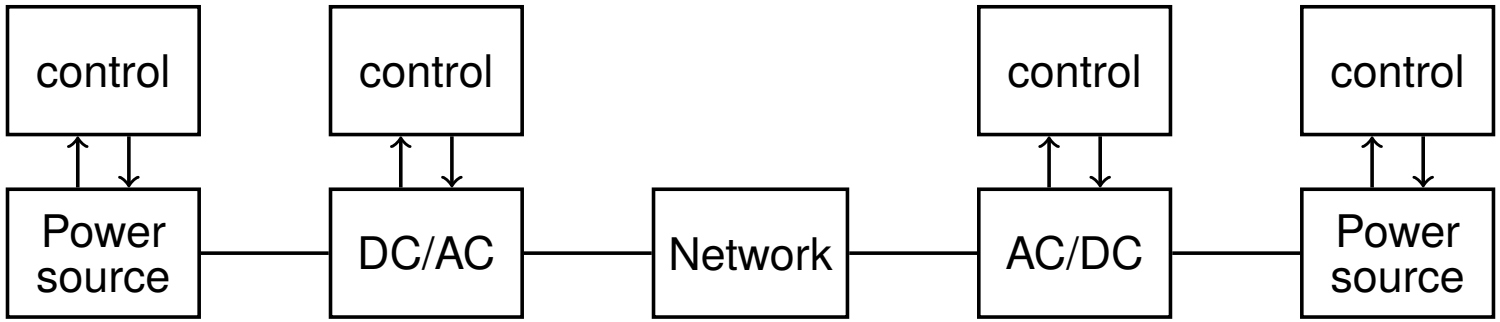


complete system model

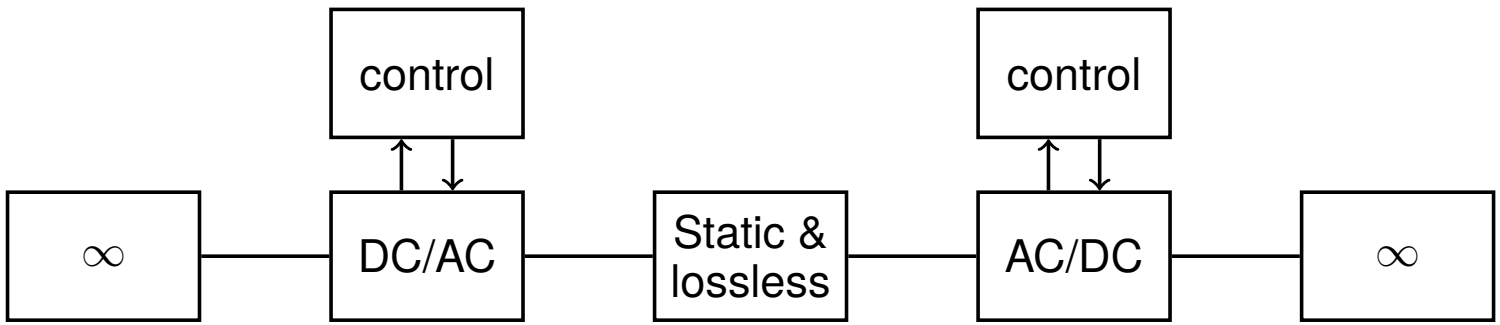


abstraction & objectives of previous controllers

Power sources & signal transformers

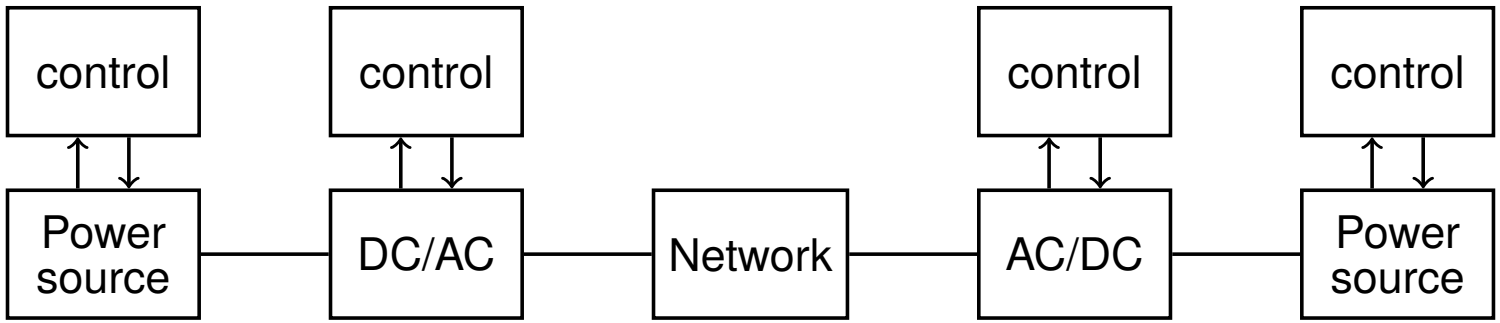


complete system model

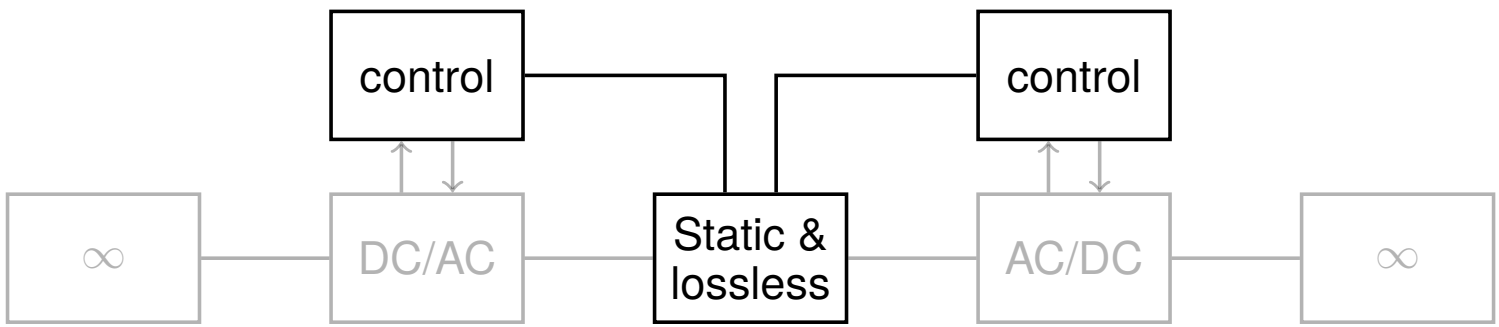


abstraction & objectives of previous controllers

Power sources & signal transformers

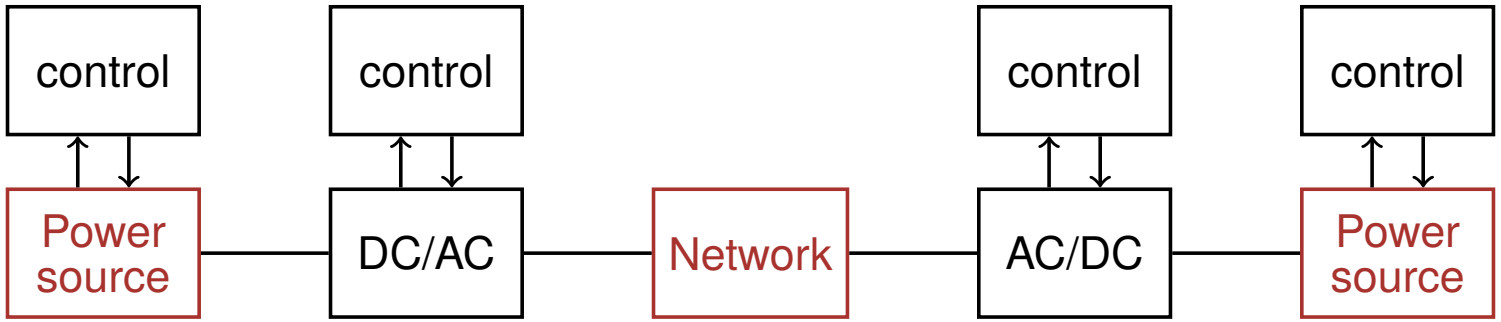


complete system model



abstraction & objectives of previous controllers

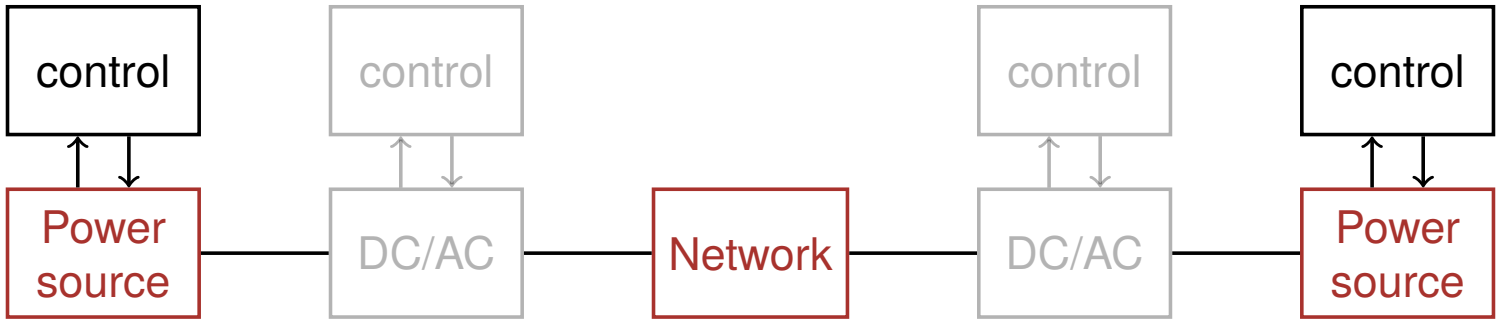
Power sources & signal transformers cont'd



power source

- governs **system-level behavior**
- response **time**, **power**, . . .

Power sources & signal transformers cont'd



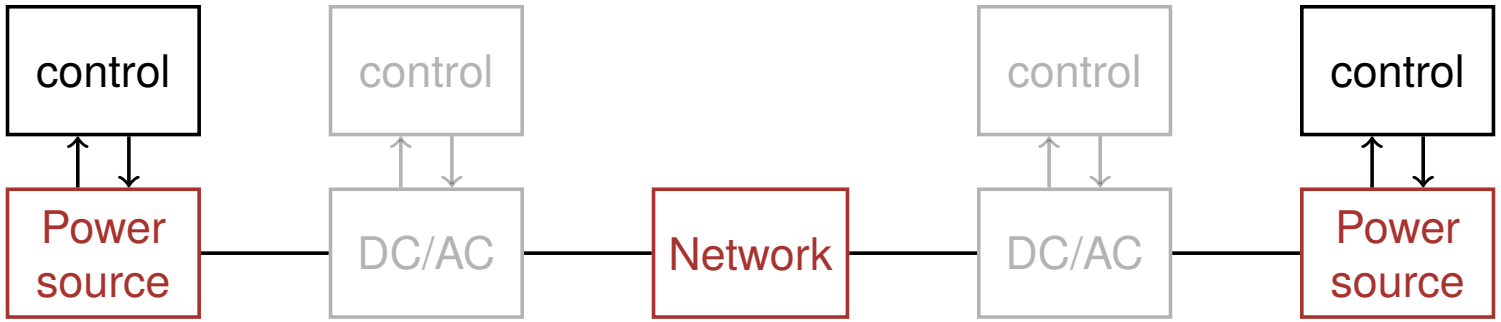
power source

- governs **system-level behavior**
- response **time**, **power**, . . .

DC/AC converter

- acts as signal **transformer**
- **negligible** controlled dynamics

Power sources & signal transformers cont'd



power source

- governs **system-level behavior**
- response **time**, **power**, . . .

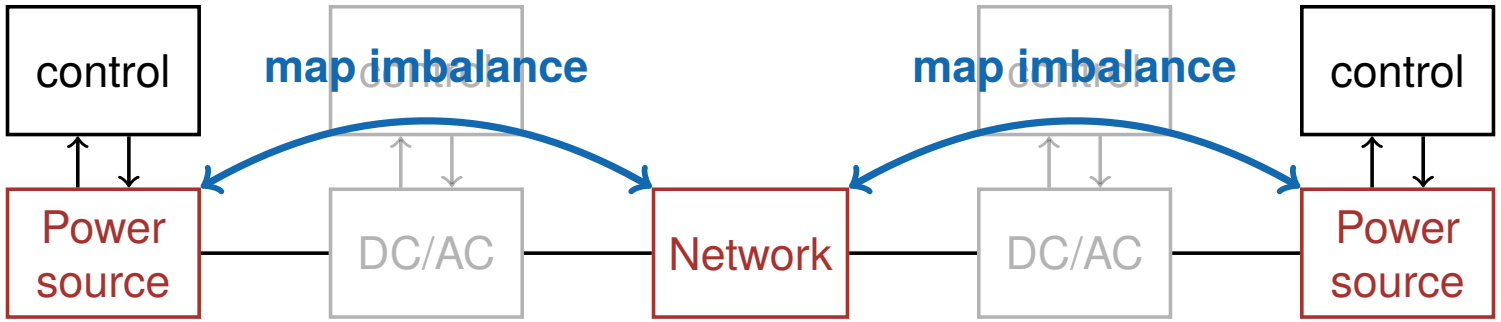
DC/AC converter

- acts as signal **transformer**
- **negligible** controlled dynamics

Key insights

- DC side & power source cannot be neglected
- focus on main **energy storage** elements & **energy source**
- synchronous machine **maps** power **imbalance** to **turbine** & **governor**

Power sources & signal transformers cont'd



power source

- governs **system-level behavior**
- response **time**, **power**, . . .

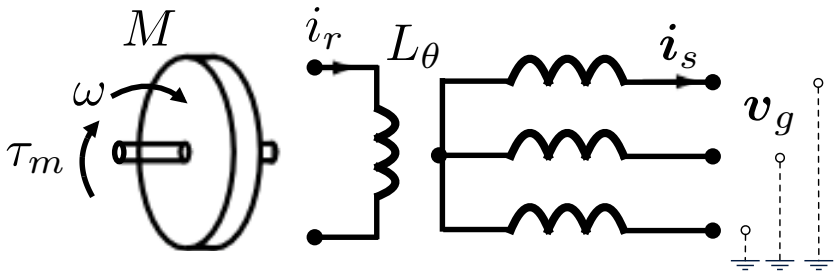
DC/AC converter

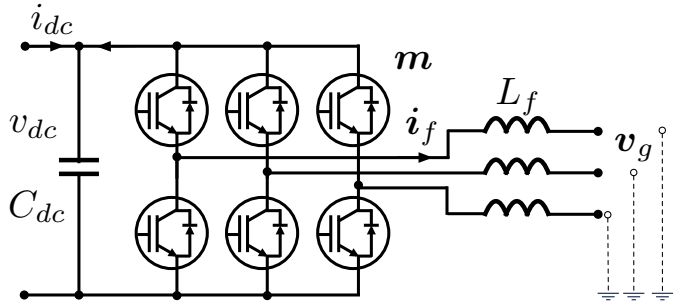
- acts as signal **transformer**
- **negligible** controlled dynamics

Key insights

- DC side & power source cannot be neglected
- focus on main **energy storage** elements & **energy source**
- synchronous machine **maps** power **imbalance** to **turbine** & **governor**
- converter maps imbalance power to . . . ?

Seeking more natural control strategies





$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega$$

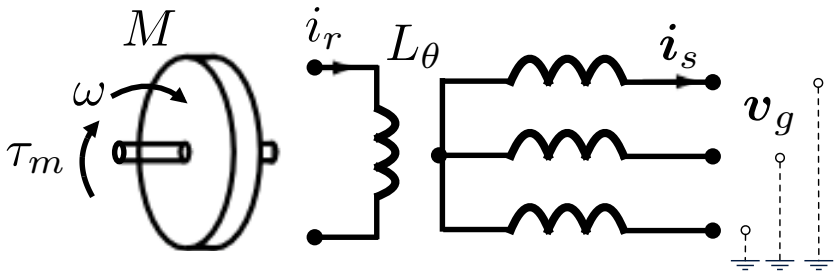
$$M\frac{\mathrm{d}\omega}{\mathrm{d}t} = -D\omega + \tau_m + L_\mathrm{m} i_r \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}^\top \boldsymbol{i}_s$$

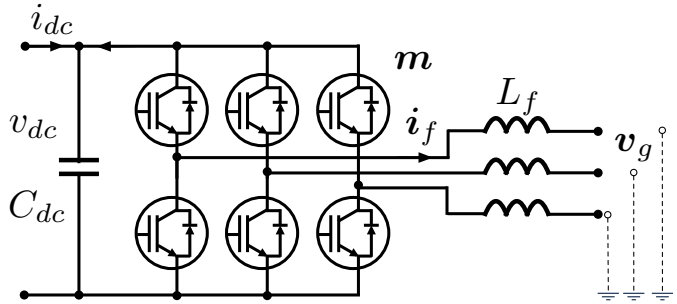
$$L_\mathrm{s}\frac{\mathrm{d}\boldsymbol{i}_s}{\mathrm{d}t} = -R_s\boldsymbol{i}_s + \boldsymbol{v}_g - L_\mathrm{m} i_r \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}\omega$$

$$C_\mathrm{dc}\frac{\mathrm{d}v_\mathrm{dc}}{\mathrm{d}t} = -G_\mathrm{dc}v_\mathrm{dc} + i_\mathrm{dc} + \boldsymbol{m}^\top \boldsymbol{i}_f$$

$$L_f\frac{\mathrm{d}\boldsymbol{i}_f}{\mathrm{d}t} = -R_f\boldsymbol{i}_f + \boldsymbol{v}_g - \boldsymbol{m}\, v_\mathrm{dc}$$

Seeking more natural control strategies





$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega$$

$$M\frac{\mathrm{d}\omega}{\mathrm{d}t} = -D\omega + \tau_m + L_\mathsf{m} i_r \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}^\top \boldsymbol{i}_s$$

$$L_\mathsf{s}\frac{\mathrm{d}\boldsymbol{i}_s}{\mathrm{d}t} = -R_s\boldsymbol{i}_s + \boldsymbol{v}_g - L_\mathsf{m} i_r \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}\omega$$

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = m_\mathsf{freq} = v_{dc} \cdot c$$

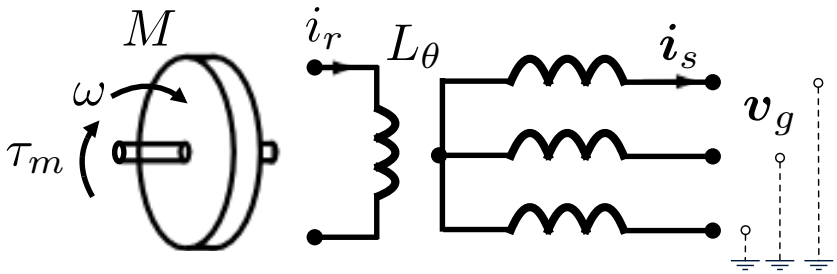
$$C_\mathsf{dc}\frac{\mathrm{d}v_\mathsf{dc}}{\mathrm{d}t} = -G_\mathsf{dc}v_\mathsf{dc} + i_\mathsf{dc} + m_\mathsf{ampl}\begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix}^\top \boldsymbol{i}_f$$

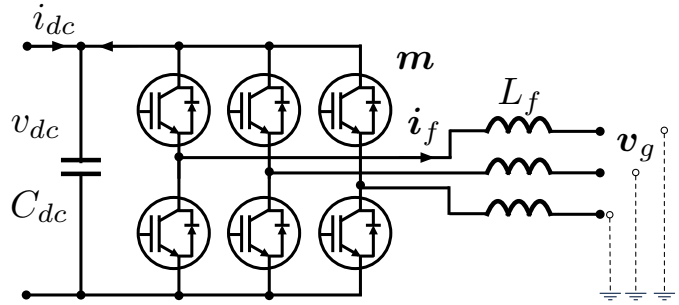
$$L_f\frac{\mathrm{d}\boldsymbol{i}_f}{\mathrm{d}t} = -R_f\boldsymbol{i}_f + \boldsymbol{v}_g - m_\mathsf{ampl}\begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix} v_\mathsf{dc}$$

1. modulation in polar coordinates:

$$\boldsymbol{m} = m_\mathsf{ampl}\begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix} \quad \& \quad \dot{\delta} = m_\mathsf{freq}$$

Seeking more natural control strategies





$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega$$

$$M\frac{\mathrm{d}\omega}{\mathrm{d}t} = -D\omega + \tau_m + L_\mathrm{m} i_r \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}^\top \boldsymbol{i}_s$$

$$L_\mathrm{s}\frac{\mathrm{d}\boldsymbol{i}_s}{\mathrm{d}t} = -R_s\boldsymbol{i}_s + \boldsymbol{v}_g - L_\mathrm{m} i_r \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}\omega$$

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = \eta \cdot v_\mathrm{dc}$$

$$C_\mathrm{dc}\frac{\mathrm{d}v_\mathrm{dc}}{\mathrm{d}t} = -G_\mathrm{dc}v_\mathrm{dc} + i_\mathrm{dc} + m_\mathrm{ampl}\begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix}^\top \boldsymbol{i}_f$$

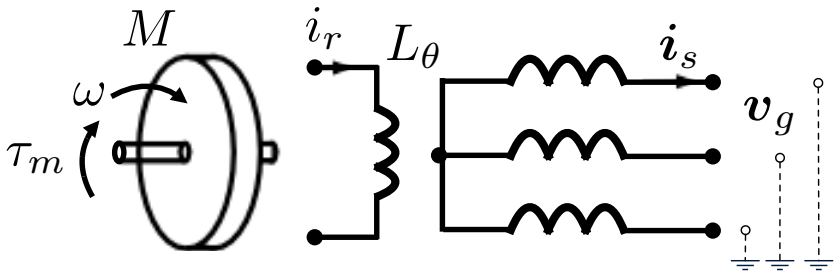
$$L_f\frac{\mathrm{d}\boldsymbol{i}_f}{\mathrm{d}t} = -R_f\boldsymbol{i}_f + \boldsymbol{v}_g - m_\mathrm{ampl}\begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix} v_\mathrm{dc}$$

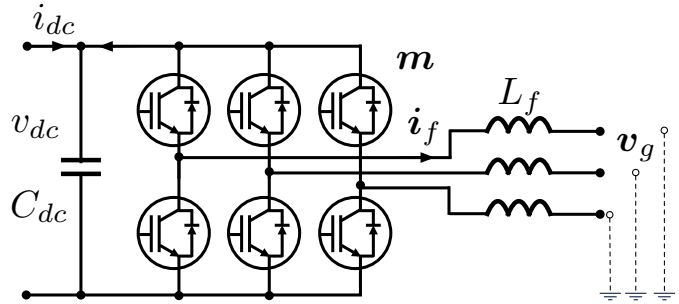
1. modulation in polar coordinates:

$$\boldsymbol{m} = m_\mathrm{ampl}\begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix} \quad \& \quad \dot{\delta} = m_\mathrm{freq}$$

2. **matching**: $m_\mathrm{freq} = \eta v_\mathrm{dc}$ with $\eta = \frac{\omega_\mathrm{ref}}{v_\mathrm{dc,ref}}$

Seeking more natural control strategies





$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega$$

$$M\frac{\mathrm{d}\omega}{\mathrm{d}t} = -D\omega + \tau_m + L_\mathrm{m} i_r \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}^\top \boldsymbol{i}_s$$

$$L_\mathrm{s}\frac{\mathrm{d}\boldsymbol{i}_s}{\mathrm{d}t} = -R_s\boldsymbol{i}_s + \boldsymbol{v}_g - L_\mathrm{m} i_r \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}\omega$$

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = \eta \cdot v_\mathrm{dc}$$

$$C_\mathrm{dc}\frac{\mathrm{d}v_\mathrm{dc}}{\mathrm{d}t} = -G_\mathrm{dc}v_\mathrm{dc} + i_\mathrm{dc} + m_\mathrm{ampl}\begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix}^\top \boldsymbol{i}_f$$

$$L_f\frac{\mathrm{d}\boldsymbol{i}_f}{\mathrm{d}t} = -R_f\boldsymbol{i}_f + \boldsymbol{v}_g - m_\mathrm{ampl}\begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix} v_\mathrm{dc}$$

1. modulation in polar coordinates:

$$\boldsymbol{m} = m_\mathrm{ampl}\begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix} \quad \& \quad \dot{\delta} = m_\mathrm{freq}$$

2. **matching**: $m_\mathrm{freq} = \eta v_\mathrm{dc}$ with $\eta = \frac{\omega_\mathrm{ref}}{v_\mathrm{dc,ref}}$

→ **duality**: $C_\mathrm{dc} \sim M$ is equivalent inertia

structural similarities:

- states: $\theta = \delta$, $\omega = \eta v_\mathrm{dc}$, $\boldsymbol{i}_s = \boldsymbol{i}_f$
- control: $u_\mathrm{ampl} = L_m i_r$, $i_\mathrm{dc}/\eta = \tau_m$

→ equivalent inertia: $M \equiv C_\mathrm{dc}/\eta^2$ & energy imbalance signal $\omega \equiv v_{dc}$

Properties of matching control

(covered on the board)

closed loop:

$$\dot{\delta} = \eta \cdot v$$

$$C\dot{v} = -Gv + i_{dc} - m_{ampl} \cdot \begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix}^T \cdot i$$

$i_{dc} = -k \cdot (v - v_{ref}) + i_{dc,ref}$

$$L\dot{i} = -Ri - v_g + m_{ampl} \begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix}$$

transform to a dq frame with angle δ:

$$\dot{\delta} = \eta v \quad \leftarrow \text{irrelevant for analysis}$$

$$C\dot{v} = -(G+k)(v - v_{ref}) + i_{dc,ref} - G v_{ref} - m_{ampl} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T i$$

$$L\dot{i} = -(R + \eta v L) i - v_g - m_{ampl} \cdot v \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

assume: $\exists$ an equilibrium $(\delta, v, i) = (\delta^*, v_{ref}, i^*)$

(δ^*: unknown; i^*)

$\rightsquigarrow$ introduce error coordinates

$$\hat{v} = v - v_{ref}, \quad \hat{i} = i - i^*$$

error dynamics:

$$C\dot{\tilde{v}} = -(G+K)\tilde{v} + m_{ampl}\begin{bmatrix}0\\ I\end{bmatrix}^T \tilde{i}$$

$$L\dot{\tilde{i}} = -(R+\eta v^* L J)\tilde{i} - m_{ampl}\tilde{v}\begin{bmatrix}0\\ I\end{bmatrix} - \eta\tilde{v} L J i^* - \eta \bar{v} L J \tilde{i}$$

Lyapunov analysis: $V = \frac{1}{2}\tilde{v}^2 C + \frac{1}{2}\|\tilde{i}\|^2 L$

$$\dot{V} = -(G+K)\tilde{v}^2 - R\|\tilde{i}\|^2 - \tilde{i}(\eta v L) J i^*$$

$$= -\begin{bmatrix}\tilde{v}\\ \tilde{i}\end{bmatrix}^T \underbrace{\begin{bmatrix} G+K & \frac{1}{2}\eta L J i^* \\ * & R \end{bmatrix}}_{Q} \begin{bmatrix}\tilde{v}\\ \tilde{i}\end{bmatrix}$$

P-control on the dc-side

Asympt. stab. $\Leftrightarrow$ Q is pos. def. $\Leftrightarrow$ $(G+K)R > \frac{L^2\eta^2}{4}\|i^*\|^2$

Properties of matching control

- simple & robust **implementation**: $v_{\text{dc}} \longrightarrow \boxed{\frac{\eta}{s}} \xrightarrow{\delta} \boxed{m_{\text{ampl}} \begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix}} \longrightarrow \boldsymbol{m}$

Properties of matching control

- simple & robust **implementation**: $v_{\text{dc}} \longrightarrow \boxed{\frac{\eta}{s}} \xrightarrow{\delta} \boxed{m_{\text{ampl}} \begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix}} \longrightarrow \boldsymbol{m}$
- exploits **structural similarities** & DC/AC **energy imbalance**
  - clarifies impact of DC side dynamics & limitations
  - saturating DC current $\rightarrow$ saturating AC current
    (no need for virtual impedance etc.)

Properties of matching control

- simple & robust **implementation**: $v_{\text{dc}} \longrightarrow \boxed{\frac{\eta}{s}} \xrightarrow{\delta} \boxed{m_{\text{ampl}} \begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix}} \longrightarrow \boldsymbol{m}$
- exploits **structural similarities** & DC/AC **energy imbalance**
  - clarifies impact of DC side dynamics & limitations
  - saturating DC current $\rightarrow$ saturating AC current
    (no need for virtual impedance etc.)
- can also be derived from **principled nonlinear control**
  - virtual angle + matching $=$ internal model + passive interconnection (IDA-PBC)

Properties of matching control

- simple & robust **implementation**: $v_{\text{dc}} \longrightarrow \boxed{\frac{\eta}{s}} \xrightarrow{\delta} \boxed{m_{\text{ampl}} \begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix}} \longrightarrow \boldsymbol{m}$
- exploits **structural similarities** & DC/AC **energy imbalance**
  - clarifies impact of DC side dynamics & limitations
  - saturating DC current $\rightarrow$ saturating AC current
    (no need for virtual impedance etc.)
- can also be derived from **principled nonlinear control**
  - virtual angle + matching $=$ internal model + passive interconnection (IDA-PBC)
- **energy shaping** via i_{dc} & u_{mag} to achieve further control objectives
  - damping/droop assignment (stability)
  - tracking set-points (PQ or PV)
  - add virtual capacitance (inertia)

Properties of matching control

- simple & robust **implementation**: $v_{\text{dc}} \longrightarrow \boxed{\frac{\eta}{s}} \xrightarrow{\delta} \boxed{m_{\text{ampl}} \begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix}} \longrightarrow \boldsymbol{m}$
- exploits **structural similarities** & DC/AC **energy imbalance**
  - clarifies impact of DC side dynamics & limitations
  - saturating DC current $\rightarrow$ saturating AC current
    (no need for virtual impedance etc.)
- can also be derived from **principled nonlinear control**
  - virtual angle + matching $=$ internal model + passive interconnection (IDA-PBC)
- **energy shaping** via i_{dc} & u_{mag} to achieve further control objectives
  - damping/droop assignment (stability)
  - tracking set-points (PQ or PV)
  - add virtual capacitance (inertia)
- closed-loop **certificates**: incremental stability & passivity $\approx$ energy decreasing

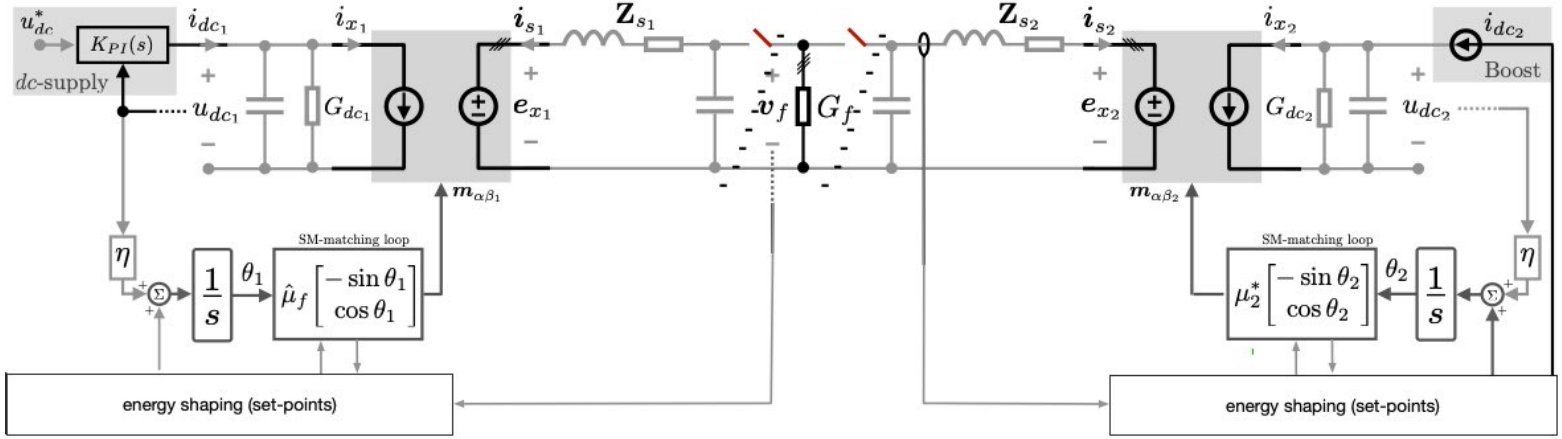
Properties of matching control

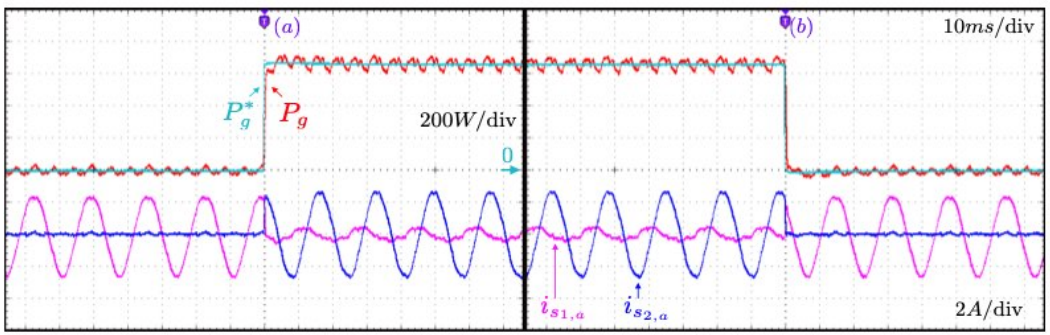
- simple & robust **implementation**: $v_{\text{dc}} \longrightarrow \boxed{\frac{\eta}{s}} \xrightarrow{\delta} \boxed{m_{\text{ampl}} \begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix}} \longrightarrow \boldsymbol{m}$
- exploits **structural similarities** & DC/AC **energy imbalance**
  - clarifies impact of DC side dynamics & limitations
  - saturating DC current $\rightarrow$ saturating AC current
    (no need for virtual impedance etc.)
- can also be derived from **principled nonlinear control**
  - virtual angle + matching $=$ internal model + passive interconnection (IDA-PBC)
- **energy shaping** via i_{dc} & u_{mag} to achieve further control objectives
  - damping/droop assignment (stability)
  - tracking set-points (PQ or PV)
  - add virtual capacitance (inertia)
- closed-loop **certificates**: incremental stability & passivity $\approx$ energy decreasing
- closed loop is **only structurally equivalent** to synchronous machine
  $\rightarrow$ time constants are different & control gains need to be tuned differently

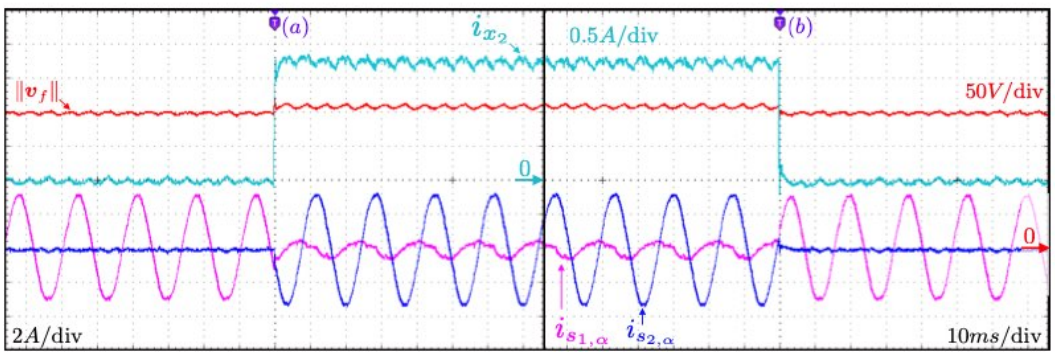
Properties of matching control

- simple & robust **implementation**: $v_{\text{dc}} \longrightarrow \boxed{\frac{\eta}{s}} \xrightarrow{\delta} \boxed{m_{\text{ampl}} \begin{bmatrix} -\sin\delta \\ \cos\delta \end{bmatrix}} \longrightarrow \boldsymbol{m}$
- exploits **structural similarities** & DC/AC **energy imbalance**
  - clarifies impact of DC side dynamics & limitations
  - saturating DC current $\rightarrow$ saturating AC current
    (no need for virtual impedance etc.)
- can also be derived from **principled nonlinear control**
  - virtual angle + matching $=$ internal model + passive interconnection (IDA-PBC)
- **energy shaping** via i_{dc} & u_{mag} to achieve further control objectives
  - damping/droop assignment (stability)
  - tracking set-points (PQ or PV)
  - add virtual capacitance (inertia)
- closed-loop **certificates**: incremental stability & passivity $\approx$ energy decreasing
- closed loop is **only structurally equivalent** to synchronous machine
  $\rightarrow$ time constants are different & control gains need to be tuned differently
- **robust but slow ac performance** $\leftarrow$ ac side controlled via dc signals

Rapid prototyping experimental validation









Details & initial conditions for further reading

The Electronic Realization of Synchronous Machines: Model Matching, Angle Tracking, and Energy Shaping Techniques

Catalin Arghir and Florian Dörfler

***Abstract*—In this article, we investigate grid-forming and grid-following control strategies starting from a nonlinear state-space modeling viewpoint. An electronic synchronous machine is an inverter whose integral of the dc-bus measurement generates the angle of the instantaneous modulation vector. We show how this minimal augmentation constitutes an exact physical realization**

running in the feedback path, which settle at the appropriate steady state. However, the large number of states of the inner loops makes the analysis of multiple converters difficult [4], motivating the study of more direct control approaches, [5], [6].

With the growing complexity of the power system at large

Grid-forming control for power converters based on matching of synchronous machines☆,☆☆

Catalin Arghir *, Taouba Jouini, Florian Dörfler

Automatic Control Laboratory at the Swiss Federal Institute of Technology (ETH) Zürich, Switzerland

ARTICLE INFO

Article history:
Received 28 June 2017
Received in revised form 16 January 2018
Accepted 20 April 2018
Available online 4 June 2018

ABSTRACT

We consider the problem of grid-forming control of power converters in low-inertia power systems. Starting from an average-switch three-phase power converter model, we draw parallels to a synchronous machine (SM) model and propose a novel converter control strategy which dwells upon the main characteristic of a SM: the presence of an internal rotating magnetic field. In particular, we augment the converter system with a virtual oscillator whose frequency is driven by the DC-side voltage measurement and which sets the converter pulse-width-modulation signal, thereby achieving exact matching between the converter in closed-loop and the SM dynamics. We then provide a sufficient condition asserting existence, uniqueness, and global asymptotic stability of a shifted equilibrium, all in a rotating coordinate frame attached to the virtual oscillator angle. By actuating the DC-side input of the converter we are able to enforce this condition and provide additional inertia and damping. In this framework, we illustrate strict incremental passivity, droop, and power-sharing properties which are compatible with conventional power system operation requirements. We subsequently adopt disturbance-decoupling and droop techniques to design additional control loops that regulate the DC-side voltage, as well as AC-side frequency and amplitude, while in the end evaluating them with numerical experiments.

© 2018 Elsevier Ltd. All rights reserved.

Also applies in a *dual-port* setup

apply matching control everywhere, i.e., to all dc/ac converters (at HVDCs, wind turbines, . . .) à la $\dot{\theta} = c_1 \cdot (p_{\text{dc}} - p^\star_{\text{dc}}) + c_2 \cdot (p_{\text{ac}} - p^\star_{\text{ac}})$ (or variations)

to **map imbalances across** & assure **dc & ac grid-forming** functionalities

Also applies in a *dual-port* setup

apply matching control everywhere, i.e., to all dc/ac converters (at HVDCs, wind turbines, . . .) à la $\dot{\theta} = c_1 \cdot (p_{\text{dc}} - p^\star_{\text{dc}}) + c_2 \cdot (p_{\text{ac}} - p^\star_{\text{ac}})$ (or variations)

to **map imbalances across** & assure **dc & ac grid-forming** functionalities

$\longrightarrow$ to get literature search started:

Dual-port grid-forming control of MMCs and its applications to grids of grids

Dominic Groß, *Member, IEEE,* Enric Sánchez-Sánchez, *Member, IEEE,* Eduardo Prieto-Araujo, *Senior Member, IEEE,* and Oriol Gomis-Bellmunt, *Fellow, IEEE*

***Abstract*—This work focuses on grid-forming (GFM) control of Interconnecting Power Converters (IPCs) that are used to interconnect multiple HVAC and HVDC subgrids to form a grid of grids. We introduce the concept of dual-port GFM control that leverages the ability of Modular Multilevel Converters (MMCs) to simultaneously form its AC and DC terminal voltage and present two dual-port GFM MMC controls. We provide analytical results and high-fidelity simulations that demonstrate that (i) dual-port GFM control is more resilient to contingencies (i.e., line and generator outages) than state-of-the-art single-port GFM control, and (ii) unlike single-port GFM control, dual-port GFM control does not require assigning grid-forming and grid-following (GFL) roles to the IPC terminals in grids of grids. Finally, we provide an in-depth discussion and comparison of single-port GFM control and the proposed dual-port GFM controls.**

grid-forming (GFM) strategies that form a stable AC voltage (i.e., magnitude and frequency) at the converter terminal. As a consequence of relying on a stable AC voltage, GFL control may fail due to voltage disturbances [4] or if insufficient GFM units (i.e., synchronous generators or GFM converters) are online to ensure frequency stability.

In contrast, GFM power converters can form a stable grid and are envisioned to be the cornerstone of future power systems. The prevalent approaches to GFM control are so-called droop-control [5], synchronous machine emulation [6], and (dispatchable) virtual oscillator control [7], [8]. All of the aforementioned controls form a stable AC voltage waveform and provide primary frequency control. However, they require

Hybrid Angle Control and Almost Global Stability of Non-synchronous Hybrid AC/DC Power Grids

Ali Tayyebi and Florian Dörfler

***Abstract*—This paper explores the stability of non-synchronous hybrid ac/dc power grids under the grid-forming hybrid angle control strategy. We formulate dynamical models for the ac grids and transmission lines, interlinking converters, and dc generations and interconnections. Next, we establish the existence and uniqueness of the closed-loop equilibria for the overall system. Subsequently, we demonstrate global attractivity of the equilibria, local asymptotic stability of the desired equilibrium point, and instability and zero-Lebesgue-measure region of attraction for other equilibria. The theoretic results are derived under mild, parametric, and unified stability/instability conditions. Finally, relying on the intermediate results, we conclude the almost global asymptotic stability of the hybrid ac/dc power grids with interlinking converters that are equipped with hybrid angle control. Last, we present several remarks on the practical and theoretical aspects of the problem under investigation.**

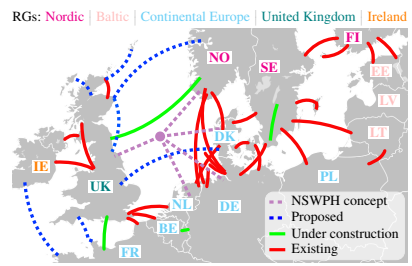


Fig. 1: The overview of the high voltage dc (HVDC) links and North Sea wind power hub (NSWPH) concept that connect the regional groups (RGs) in the Northern Europe and Baltic regions [1].

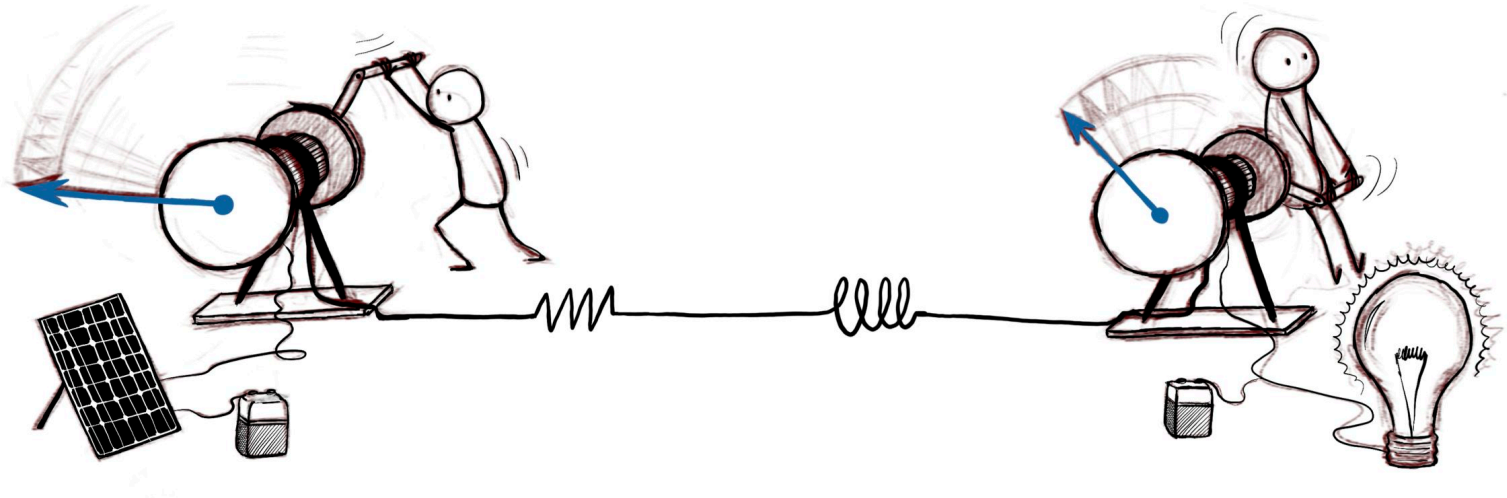
Outline

- Virtual Synchronous Machine Emulation
- Droop Control
- Matching Control
- Virtual Oscillator Control (VOC)
- Time-Scale Separation Issues
- Comparison

Cartoon summary of VOC

(figures by Outi Supponen)

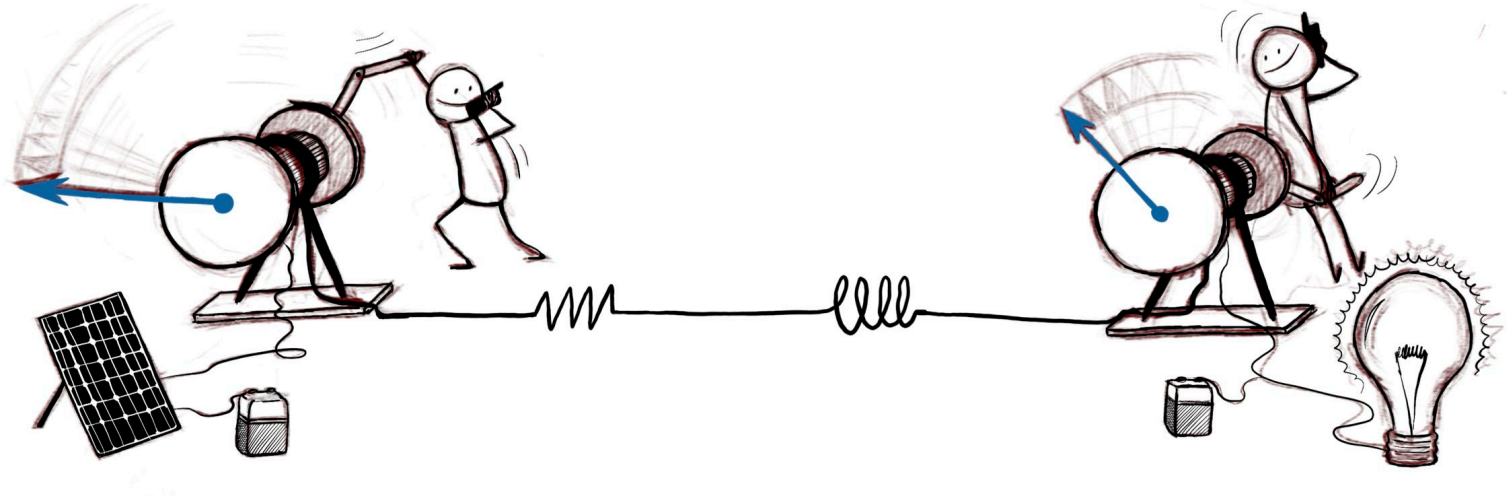
Conceptually, inverters are oscillators that have to synchronize



Cartoon summary of VOC

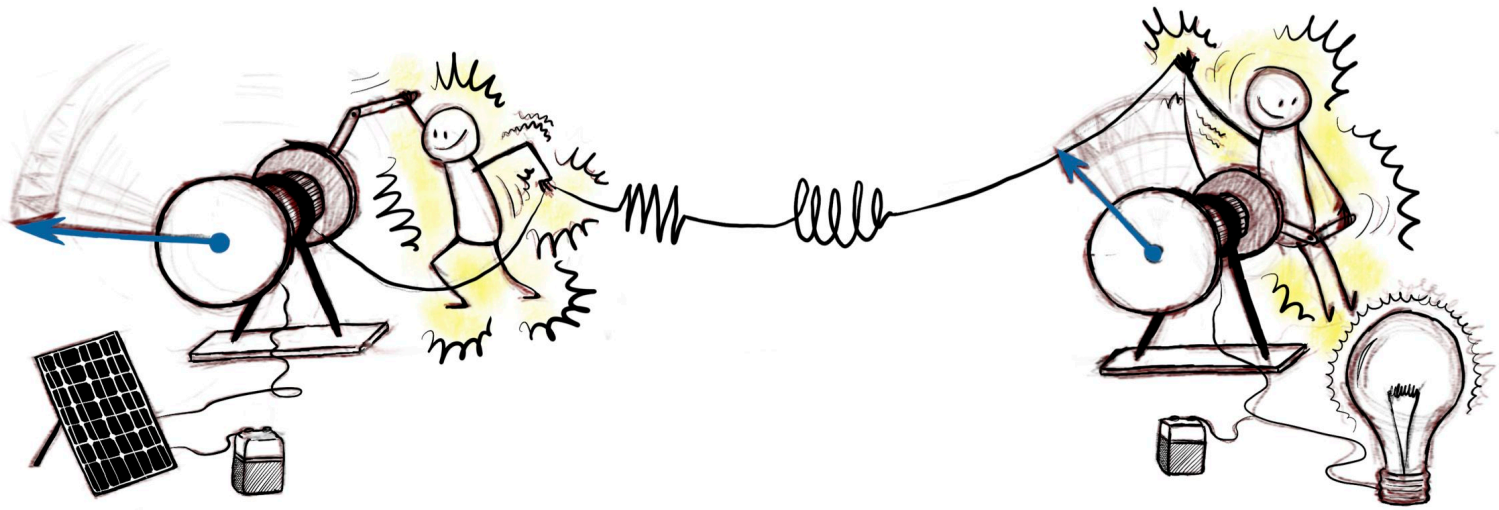
(figures by Outi Supponen)

Hypothetically, they could sync by communication (not feasible)



Cartoon summary of VOC (figures by Outi Supponen)

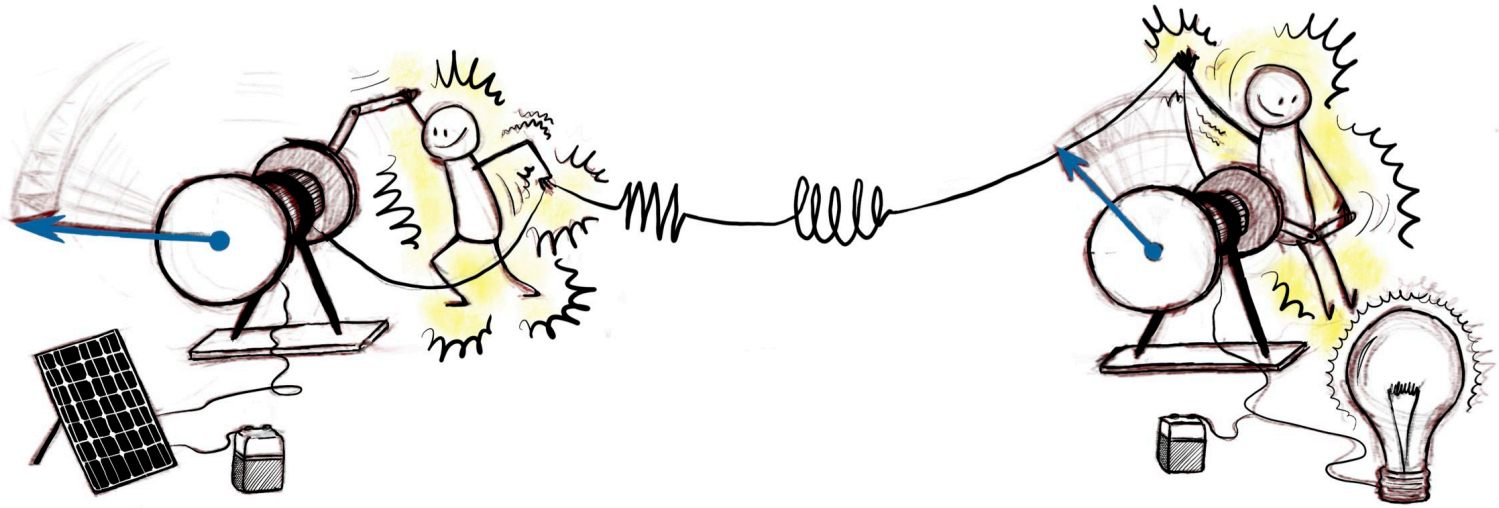
Colorful idea: inverters sync through physics & clever local control

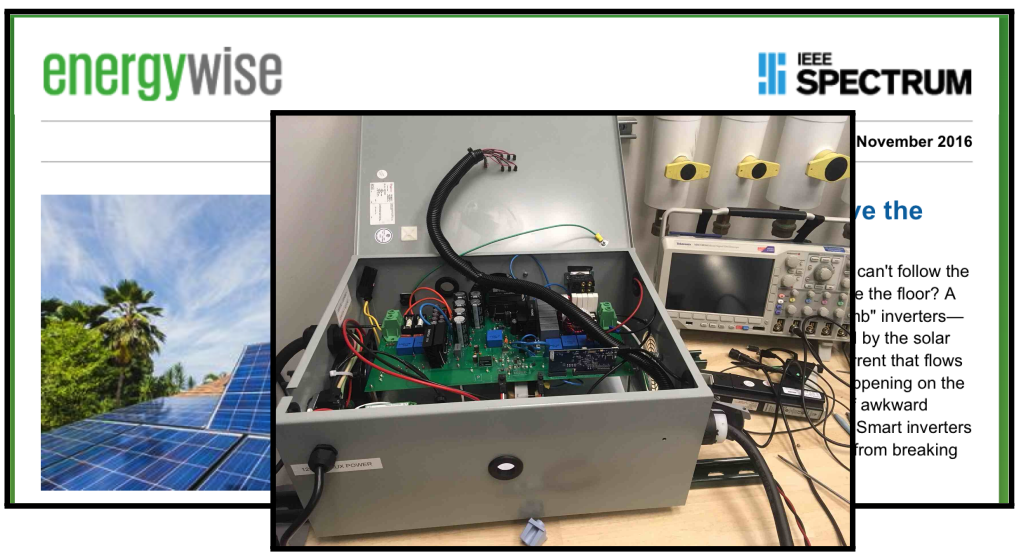


Cartoon summary of VOC

(figures by Outi Supponen)

Colorful idea: inverters sync through physics & clever local control



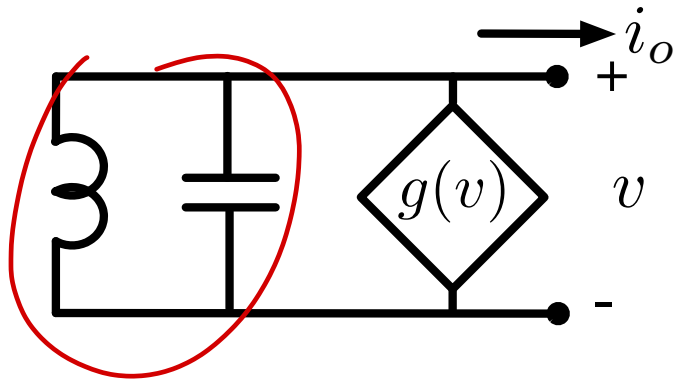


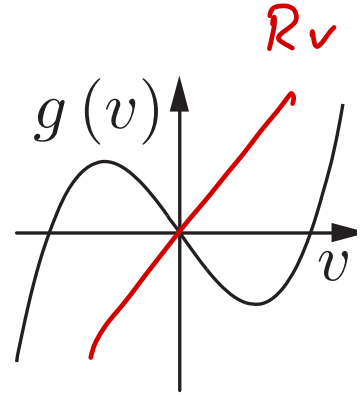
theory: sync of coupled oscillators & nonlinear decentralized control

power systems/electronics **experiments** @NREL outstanding performance (superior to droop control)

Original **V**irtual **O**scillator **C**ontrol (VOC)

nonlinear & open limit cycle oscillator as reference model



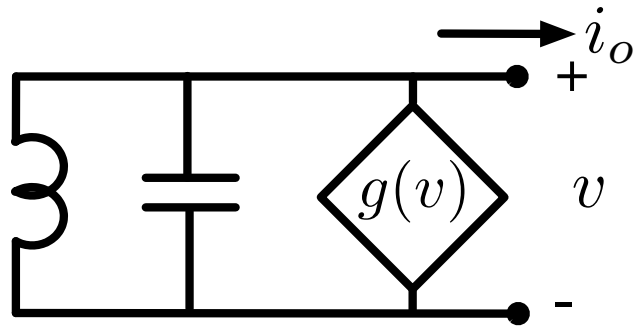


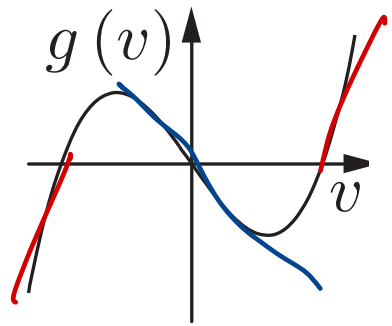
$$\left.\begin{aligned} L\dot{i} &= v \\ C\dot{v} &= -i \end{aligned}\right\} \quad C\ddot{v} = -\tfrac{1}{L}v$$

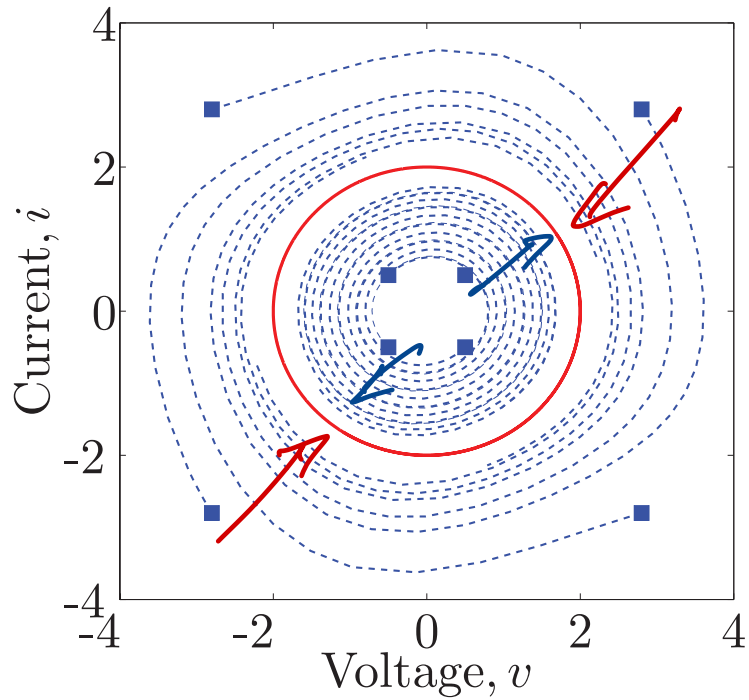
$$\rightsquigarrow v(t) \sim \sin\left(\sqrt{\tfrac{1}{LC}} \cdot t\right)$$

Original **V**irtual **O**scillator **C**ontrol (VOC)

nonlinear & open limit cycle oscillator as reference model

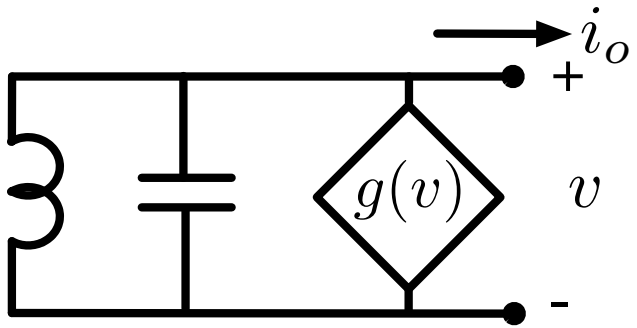


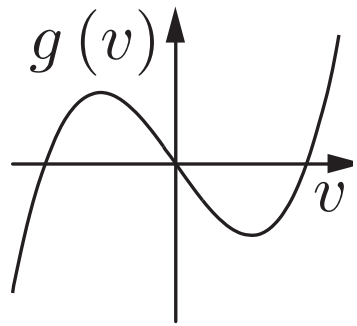




Original **V**irtual **O**scillator **C**ontrol (VOC)

nonlinear & open limit cycle oscillator as reference model





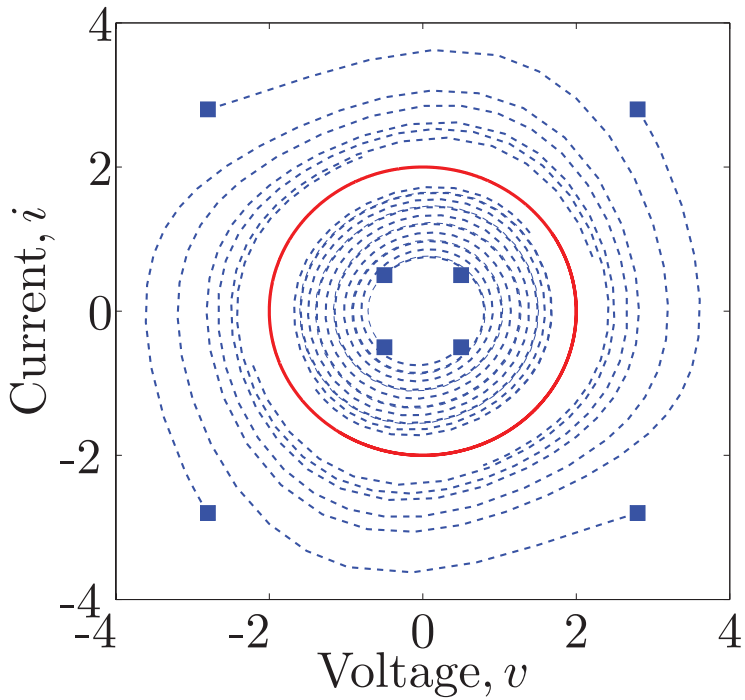
- simplified model amenable to theoretic analysis

  [J. Aracil & F. Gordillo, '02], [Torres, Hespanha, Moehlis, '11], [Johnson, Dhople, Krein, '13], [Dhople, Johnson, Dörfler, '14]

→ **almost global synchronization** & **local droop**

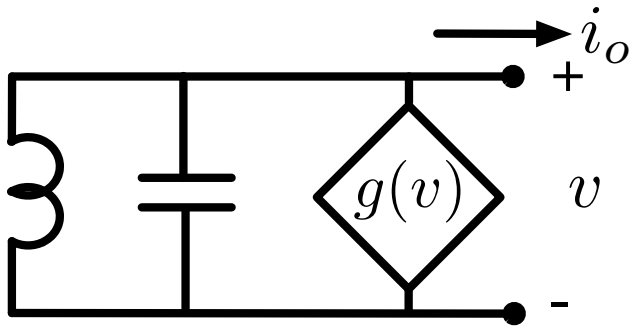
- in practice proven to be **robust mechanism** with performance superior to droop & others

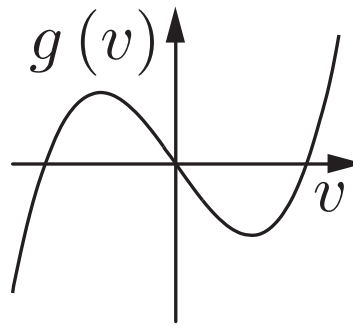
→ **problem**: cannot be controlled(?) to meet specifications on amplitude & power injections



Original **V**irtual **O**scillator **C**ontrol (VOC)

nonlinear & open limit cycle oscillator as reference model





- simplified model amenable to theoretic analysis

  [J. Aracil & F. Gordillo, '02], [Torres, Hespanha, Moehlis, '11], [Johnson, Dhople, Krein, '13], [Dhople, Johnson, Dörfler, '14]

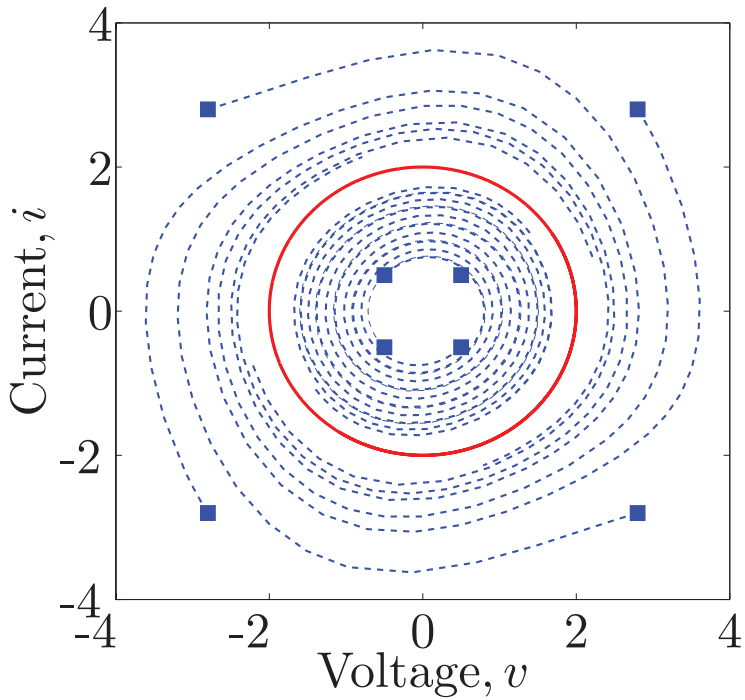
→ **almost global synchronization** & **local droop**

- in practice proven to be **robust mechanism** with performance superior to droop & others

→ **problem**: cannot be controlled(?) to meet specifications on amplitude & power injections

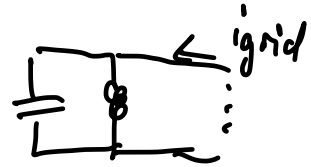
→ **dispatchable** virtual oscillator control

[Colombino, Groß, Brouillon, & Dörfler, '17, '18,'19], [Subotic, Gross, Colombino, & Dörfler,'19]



Synchronization analysis

(covered on the board)



$$L\dot{i} = v$$

$$C\dot{v} = -i - i_{grid}$$

Linear synchronization:

coupling through a resistive network:

$$\begin{bmatrix} \vdots \\ i_{grid,i} \\ \vdots \end{bmatrix} = L \cdot \begin{bmatrix} \vdots \\ v_i \\ \vdots \end{bmatrix}$$

↳ interconnected dynamics:

$$L\dot{i} = v$$

$$C\dot{v} = -i - Lv$$

Laplacian

$$i_{grid,i} = \sum_j a_{ij}(v_i - v_j)$$

↳ diagonalization:

$$\tilde{v} = \bar{V}v$$

$$\tilde{i} = \bar{V}i$$

$$L = \underbrace{[v_1 \dots v_n]}_{\bar{V}} \cdot \begin{bmatrix} 0 & & \\ & \lambda_2 & \\ & & \ddots \\ & & & \lambda_n \end{bmatrix} [v_1 \dots v_n]^T$$

$$L\dot{\tilde{i}} = \tilde{v}$$

$$C\dot{\tilde{v}} = -\tilde{i} - \begin{bmatrix} 0 & & \\ & \lambda_2 & \\ & & \ddots \\ & & & \lambda_n \end{bmatrix} \tilde{v}$$

mode-by-mode:

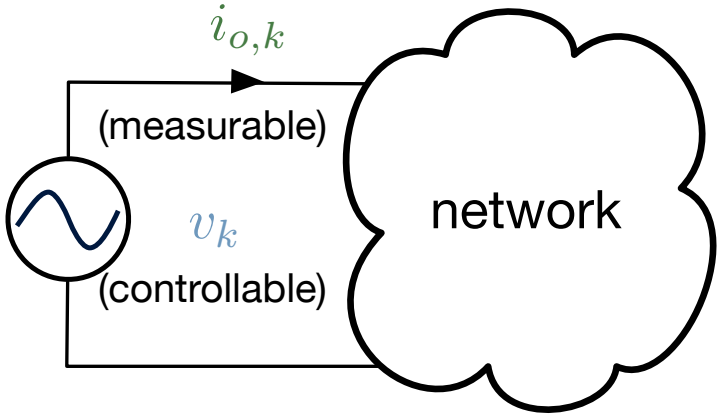
0th mode:
$$\left.\begin{aligned} L\,\dot{\tilde{i}}_0 &= \tilde{v}_0 \\ C\,\dot{\tilde{v}}_0 &= -\tilde{i}_0 \end{aligned}\right\} \rightsquigarrow \sin\left(\sqrt{\tfrac{1}{LC}}\,t\right)\ldots$$

higher-modes:
$$\left.\begin{aligned} L\,\dot{\tilde{i}}_i &= \tilde{v}_i \\ C\,\dot{\tilde{v}}_i &= -\tilde{i}_i - \lambda_i v_i \end{aligned}\right\} \text{asympt. stable} \longrightarrow 0$$

↳ original dynamics: only mode $v_1 = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ survives $\rightsquigarrow$ synchronize on sinusoidal waveforms

improvement of original ad hoc virtual oscillator control (VOC)

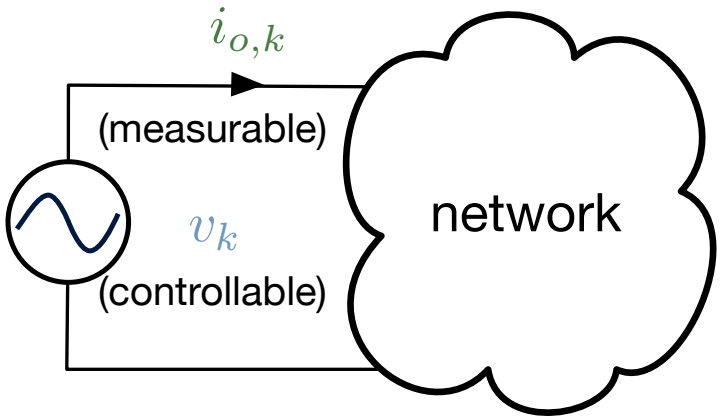
Model & control objectives (assumptions easy to generalize)



simplified multi-converter system model

- converter $=$ **terminal voltage** $v_k \in \mathbb{R}^2$

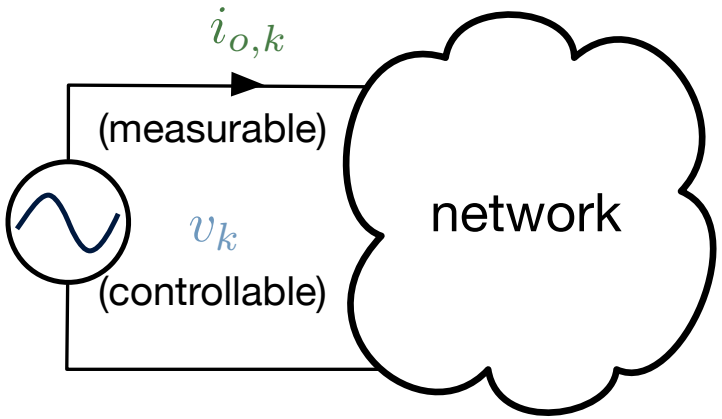
Model & control objectives (assumptions easy to generalize)



simplified multi-converter system model

- converter $=$ **terminal voltage** $v_k \in \mathbb{R}^2$
- **line dynamics** = steady-state Π-model with line admittance $\|Y_{jk}\| = 1/\sqrt{r_{kj}^2 + \omega_0^2 \ell_{kj}^2}$

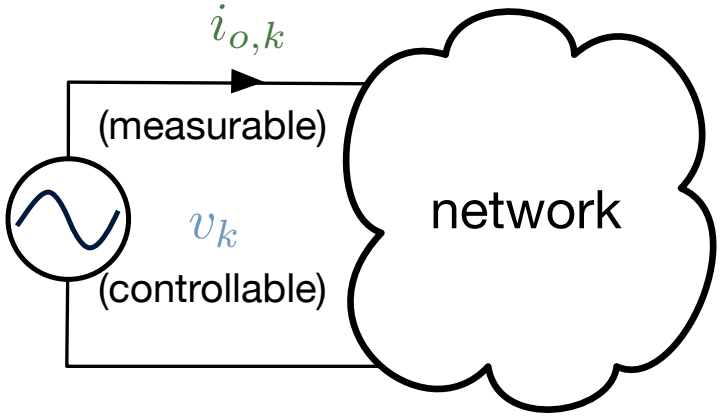
Model & control objectives (assumptions easy to generalize)



simplified multi-converter system model

- converter $=$ **terminal voltage** $v_k \in \mathbb{R}^2$
- **line dynamics** = steady-state Π-model with line admittance $\|Y_{jk}\| = 1/\sqrt{r_{kj}^2 + \omega_0^2 \ell_{kj}^2}$
- **homogeneous lines** with $\kappa = \frac{\ell_{jk}}{r_{jk}}$ constant

Model & control objectives (assumptions easy to generalize)

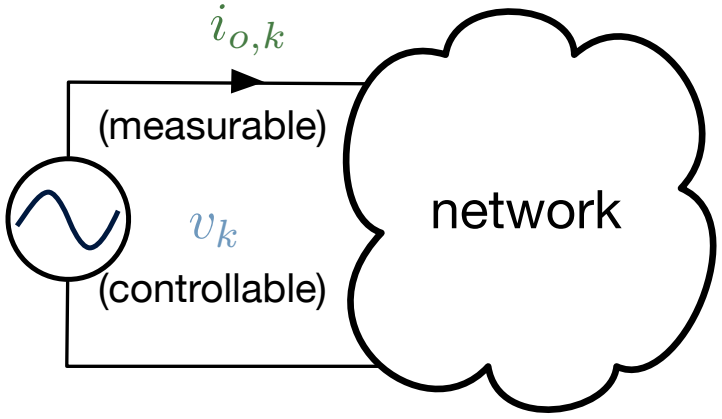


simplified multi-converter system model

- converter = **terminal voltage** $v_k \in \mathbb{R}^2$
- **line dynamics** = steady-state Π-model with line admittance $\|Y_{jk}\| = 1/\sqrt{r_{kj}^2 + \omega_0^2 \ell_{kj}^2}$
- **homogeneous lines** with $\kappa = \frac{\ell_{jk}}{r_{jk}}$ constant

desired steady-state behavior

Model & control objectives (assumptions easy to generalize)



simplified multi-converter system model

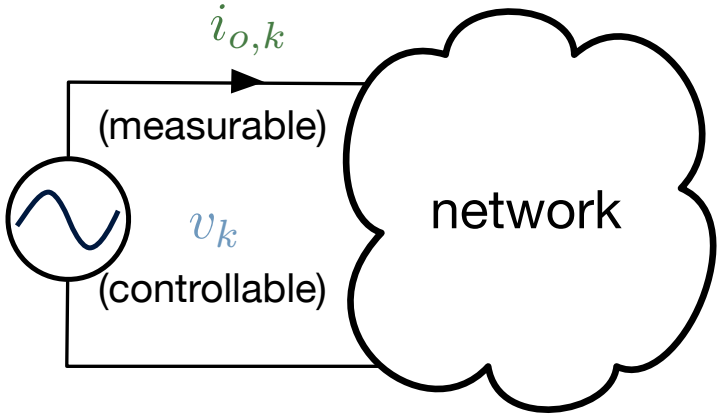
- converter = **terminal voltage** $v_k \in \mathbb{R}^2$
- **line dynamics** = steady-state Π-model with line admittance $\|Y_{jk}\| = 1/\sqrt{r_{kj}^2 + \omega_0^2 \ell_{kj}^2}$
- **homogeneous lines** with $\kappa = \frac{\ell_{jk}}{r_{jk}}$ constant

desired steady-state behavior

- nominal **synchronous frequency**

$$\frac{d}{dt} v_k = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} v_k$$

Model & control objectives (assumptions easy to generalize)



simplified multi-converter system model

- converter $=$ **terminal voltage** $v_k \in \mathbb{R}^2$
- **line dynamics** = steady-state Π-model with line admittance $\|Y_{jk}\| = 1/\sqrt{r_{kj}^2 + \omega_0^2 \ell_{kj}^2}$
- **homogeneous lines** with $\kappa = \frac{\ell_{jk}}{r_{jk}}$ constant

desired steady-state behavior

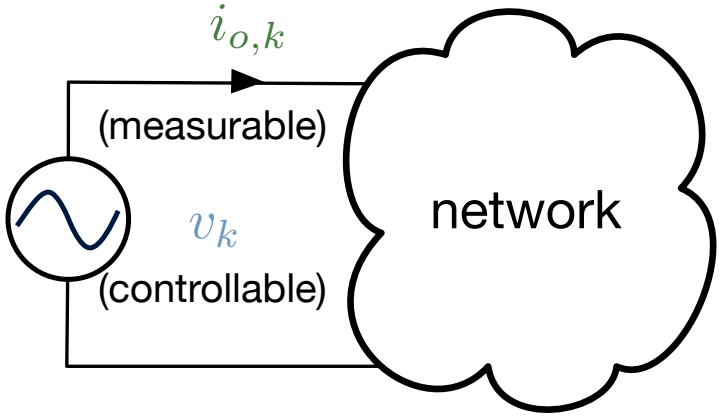
- nominal **synchronous frequency**

$$\frac{d}{dt} v_k = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} v_k$$

- voltage **amplitude** (uniform for simplicity)

$$\|v_k\| = v^\star$$

Model & control objectives (assumptions easy to generalize)



simplified multi-converter system model

- converter $=$ **terminal voltage** $v_k \in \mathbb{R}^2$
- **line dynamics** = steady-state Π-model with line admittance $\|Y_{jk}\| = 1/\sqrt{r_{kj}^2 + \omega_0^2 \ell_{kj}^2}$
- **homogeneous lines** with $\kappa = \frac{\ell_{jk}}{r_{jk}}$ constant

desired steady-state behavior

- nominal **synchronous frequency**

$$\tfrac{d}{dt}\, v_k = \left[\begin{smallmatrix} 0 & -\omega \\ \omega & 0 \end{smallmatrix}\right] v_k$$

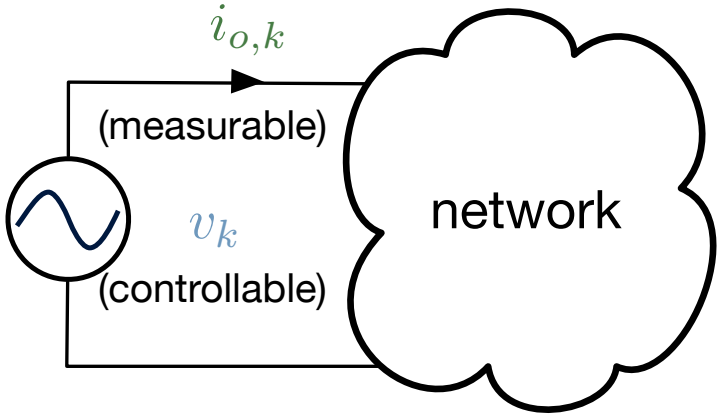
- voltage **amplitude** (uniform for simplicity)

$$\|v_k\| = v^\star$$

- active & reactive **power injection**

$$v_k^\top\, i_{o,k} = p_k^\star \quad , \quad v_k^\top \left[\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right] i_{o,k} = q_k^\star$$

Model & control objectives (assumptions easy to generalize)



simplified multi-converter system model

- converter = **terminal voltage** $v_k \in \mathbb{R}^2$
- **line dynamics** = steady-state Π-model with line admittance $\|Y_{jk}\| = 1/\sqrt{r_{kj}^2 + \omega_0^2 \ell_{kj}^2}$
- **homogeneous lines** with $\kappa = \frac{\ell_{jk}}{r_{jk}}$ constant

desired steady-state behavior

- nominal **synchronous frequency**

$$\tfrac{d}{dt}\, v_k = \left[\begin{smallmatrix} 0 & -\omega \\ \omega & 0 \end{smallmatrix}\right] v_k$$

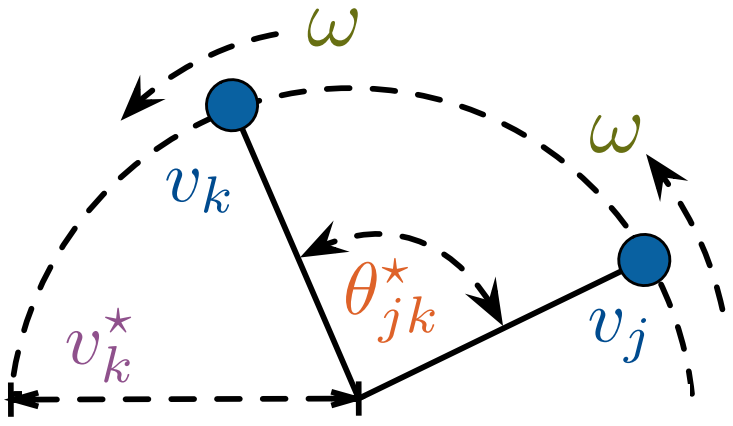
- voltage **amplitude** (uniform for simplicity)

$$\|v_k\| = v^\star$$

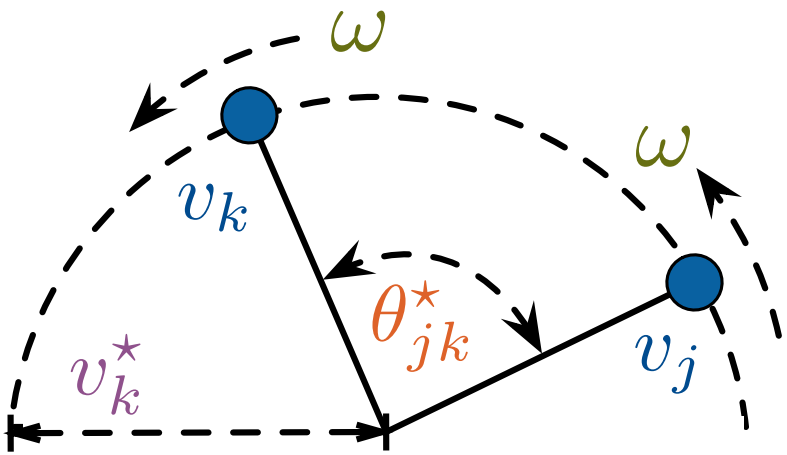
- active & reactive **power injection**

$$v_k^\top\, i_{o,k} = p_k^\star \quad , \quad v_k^\top \left[\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right] i_{o,k} = q_k^\star$$

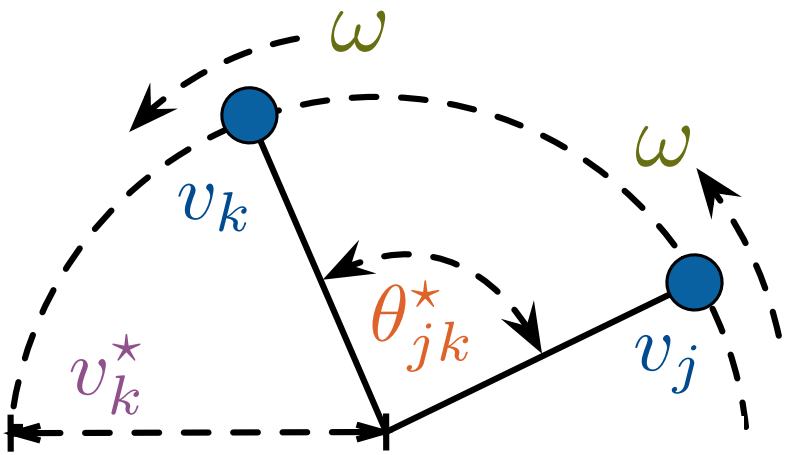
$\Leftrightarrow$ relative **angles**: $v_k = \begin{bmatrix} \cos(\theta_{jk}^\star) & -\sin(\theta_{jk}^\star) \\ \sin(\theta_{jk}^\star) & \cos(\theta_{jk}^\star) \end{bmatrix} v_j$

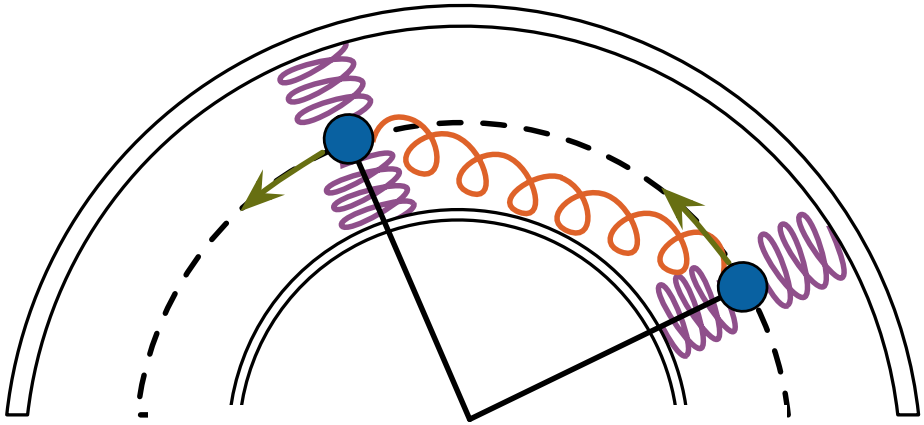


Colorful idea: closed-loop target dynamics

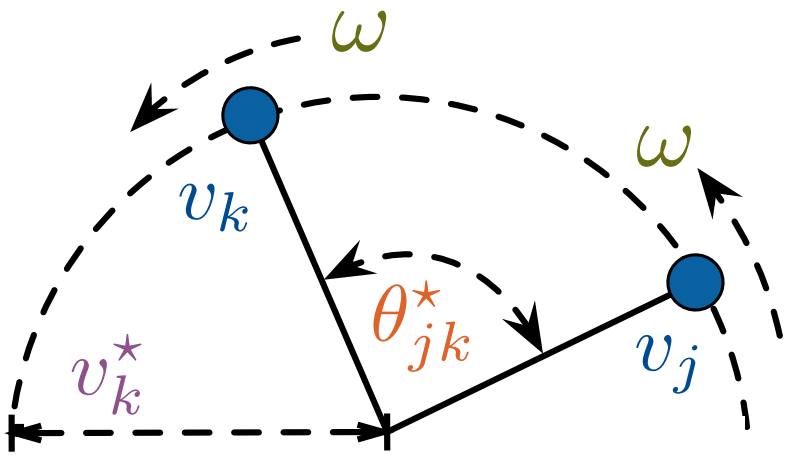


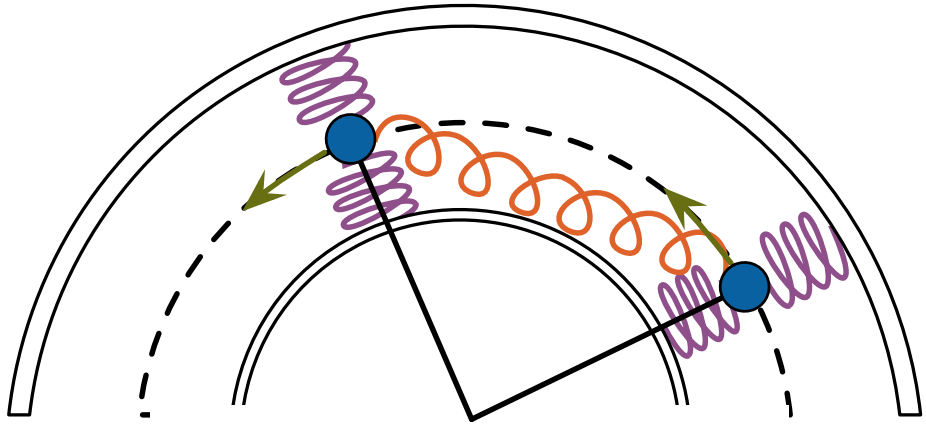
Colorful idea: closed-loop target dynamics





Colorful idea: closed-loop target dynamics





$$\frac{d}{dt}\boldsymbol{v_k} = \underbrace{\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \boldsymbol{v_k}}_{\text{rotation at } \omega} + \underbrace{c_1 \cdot \left(\|v_k\|^{\star 2} - \|\boldsymbol{v}_k\|^2 \right) \boldsymbol{v}_k}_{\text{amplitude regulation to } v_k^\star}$$

$$+ \underbrace{c_2 \cdot \sum_{j=1}^{n} w_{jk} \left(\boldsymbol{v}_j - \begin{bmatrix} \cos(\theta^\star_{jk}) & -\sin(\theta^\star_{jk}) \\ \sin(\theta^\star_{jk}) & \cos(\theta^\star_{jk}) \end{bmatrix} \boldsymbol{v}_k \right)}_{\text{synchronization to desired relative angles } \theta^\star_{jk}}$$

Decentralized implementation of dynamics

$$\underbrace{\sum_j w_{jk}(v_j - R(\theta^\star_{jk})v_k)}$$

need to know w_{jk}, v_j, v_k and $\theta^\star_{jk}$

Decentralized implementation of dynamics

$$\underbrace{\sum_j w_{jk}(v_j - R(\theta^\star_{jk})v_k)}_{\text{need to know } w_{jk},\, v_j,\, v_k \text{ and } \theta^\star_{jk}} = \underbrace{\sum_j w_{jk}(v_j - v_k)}_{\text{“Laplacian” feedback}} + \underbrace{\sum_j w_{jk}(I - R(\theta^\star_{jk}))v_k}_{\text{local feedback: } \mathcal{K}_k(\theta^\star)v_k}$$

Decentralized implementation of dynamics

$$\underbrace{\sum_j w_{jk}(v_j - R(\theta^\star_{jk})v_k)}_{\text{need to know } w_{jk},\, v_j,\, v_k \text{ and } \theta^\star_{jk}} = \underbrace{\sum_j w_{jk}(v_j - v_k)}_{\text{“Laplacian” feedback}} + \underbrace{\sum_j w_{jk}(I - R(\theta^\star_{jk}))v_k}_{\text{local feedback: } \mathcal{K}_k(\theta^\star)v_k}$$

insight I: non-local measurements from **communication via physics**

$$\underbrace{i_{o,k}}_{\text{local feedback}} = \underbrace{\sum_j y_{jk}(v_j - v_k)}_{\text{distributed feedback with } w_{jk} = y_{kj} = \|y_{kj}\|\, R(\kappa)^{-1}}$$

Decentralized implementation of dynamics

$$\underbrace{\sum_j w_{jk}(v_j - R(\theta^\star_{jk})v_k)}_{\text{need to know } w_{jk},\, v_j,\, v_k \text{ and } \theta^\star_{jk}} = \underbrace{\sum_j w_{jk}(v_j - v_k)}_{\text{“Laplacian” feedback}} + \underbrace{\sum_j w_{jk}(I - R(\theta^\star_{jk}))v_k}_{\text{local feedback: } \mathcal{K}_k(\theta^\star)v_k}$$

insight I: non-local measurements from **communication via physics**

$$\underbrace{i_{o,k}}_{\text{local feedback}} = \underbrace{\sum_j y_{jk}(v_j - v_k)}_{\text{distributed feedback with } w_{jk} = y_{kj} = \|y_{kj}\|\, R(\kappa)^{-1}}$$

insight II: angle set-points & line-parameters from **power flow equations**

$$p^\star_k = v^{\star 2} \sum_j \frac{r_{jk}(1-\cos(\theta^\star_{jk})) - \omega_0 \ell_{jk} \sin(\theta^\star_{jk})}{r_{jk}^2 + \omega_0^2 \ell_{jk}^2}$$

$$q^\star_k = -v^{\star 2} \sum_j \frac{\omega_0 \ell_{jk}(1-\cos(\theta^\star_{jk})) + r_{jk} \sin(\theta^\star_{jk})}{r_{jk}^2 + \omega_0^2 \ell_{jk}^2}$$

Decentralized implementation of dynamics

$$\underbrace{\sum_j w_{jk}(v_j - R(\theta^\star_{jk})v_k)}_{\text{need to know } w_{jk},\, v_j,\, v_k \text{ and } \theta^\star_{jk}} = \underbrace{\sum_j w_{jk}(v_j - v_k)}_{\text{“Laplacian” feedback}} + \underbrace{\sum_j w_{jk}(I - R(\theta^\star_{jk}))v_k}_{\text{local feedback: } \mathcal{K}_k(\theta^\star)v_k}$$

insight I: non-local measurements from **communication via physics**

$$\underbrace{i_{o,k}}_{\text{local feedback}} = \underbrace{\sum_j y_{jk}(v_j - v_k)}_{\text{distributed feedback with } w_{jk} = y_{kj} = \|y_{kj}\|\, R(\kappa)^{-1}}$$

$\kappa = R/L$ ratio

insight II: angle set-points & line-parameters from **power flow equations**

$$\left.\begin{aligned} p^\star_k &= v^{\star 2} \sum_j \frac{r_{jk}(1-\cos(\theta^\star_{jk})) - \omega_0 \ell_{jk} \sin(\theta^\star_{jk})}{r^2_{jk} + \omega_0^2 \ell^2_{jk}} \\ q^\star_k &= -v^{\star 2} \sum_j \frac{\omega_0 \ell_{jk}(1-\cos(\theta^\star_{jk})) + r_{jk} \sin(\theta^\star_{jk})}{r^2_{jk} + \omega_0^2 \ell^2_{jk}} \end{aligned}\right\} \Rightarrow \underbrace{\mathcal{K}_k(\theta^\star)}_{\text{global parameters}} = \underbrace{\frac{1}{v^{\star 2}} R(\kappa) \begin{bmatrix} q^\star_k & p^\star_k \\ -p^\star_k & q^\star_k \end{bmatrix}}_{\text{local parameters}}$$

Properties of virtual oscillator control

1. desired target dynamics can be realized via **fully decentralized control**

$$\frac{d}{dt} v_k = \underbrace{\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} v_k}_{\text{rotation at } \omega_0} + c_1 \cdot \underbrace{\left(v^{\star 2} - \|v_k\|^2\right) v_k}_{\text{local amplitude regulation}} + c_2 \cdot \underbrace{R(\kappa) \left(\frac{1}{v^{\star 2}} \begin{bmatrix} q_k^\star & p_k^\star \\ -p_k^\star & q_k^\star \end{bmatrix} v_k - i_{o,k} \right)}_{\text{synchronization through physics}}$$

Properties of virtual oscillator control

1. desired target dynamics can be realized via **fully decentralized control**

$$\frac{d}{dt} v_k = \underbrace{\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} v_k}_{\text{rotation at } \omega_0} + c_1 \cdot \underbrace{\left(v^{\star 2} - \|v_k\|^2\right) v_k}_{\text{local amplitude regulation}} + c_2 \cdot \underbrace{R(\kappa) \left(\frac{1}{v^{\star 2}} \begin{bmatrix} q_k^\star & p_k^\star \\ -p_k^\star & q_k^\star \end{bmatrix} v_k - i_{o,k} \right)}_{\text{synchronization through physics}}$$

2. connection to **droop control** revealed in polar coordinates (for inductive grid)

$$\frac{d}{dt} \theta_k = \omega_0 + c_1 \left(\frac{p_k^\star}{v^{\star 2}} - \frac{p_k}{\|v_k\|^2} \right)$$

Properties of virtual oscillator control

1. desired target dynamics can be realized via **fully decentralized control**

$$\frac{d}{dt}v_k = \underbrace{\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} v_k}_{\text{rotation at } \omega_0} + c_1 \cdot \underbrace{\left(v^{\star 2} - \|v_k\|^2\right) v_k}_{\text{local amplitude regulation}} + c_2 \cdot \underbrace{R(\kappa)\left(\frac{1}{v^{\star 2}} \begin{bmatrix} q_k^\star & p_k^\star \\ -p_k^\star & q_k^\star \end{bmatrix} v_k - i_{o,k}\right)}_{\text{synchronization through physics}}$$

2. connection to **droop control** revealed in polar coordinates (for inductive grid)

$$\frac{d}{dt}\theta_k = \omega_0 + c_1\left(\frac{p_k^\star}{v^{\star 2}} - \frac{p_k}{\|v_k\|^2}\right) \underset{\|v_k\|\approx 1}{\approx} \omega_0 + c_2\left(p_k^\star - p_k\right) \quad (p-\omega \text{ droop})$$

Properties of virtual oscillator control

1. desired target dynamics can be realized via **fully decentralized control**

$$\frac{d}{dt}v_k = \underbrace{\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} v_k}_{\text{rotation at } \omega_0} + c_1 \cdot \underbrace{\left(v^{\star 2} - \|v_k\|^2\right) v_k}_{\text{local amplitude regulation}} + c_2 \cdot \underbrace{R(\kappa)\left(\frac{1}{v^{\star 2}} \begin{bmatrix} q_k^\star & p_k^\star \\ -p_k^\star & q_k^\star \end{bmatrix} v_k - i_{o,k}\right)}_{\text{synchronization through physics}}$$

2. connection to **droop control** revealed in polar coordinates (for inductive grid)

$$\frac{d}{dt}\theta_k = \omega_0 + c_1\left(\frac{p_k^\star}{v^{\star 2}} - \frac{p_k}{\|v_k\|^2}\right) \underset{\|v_k\|\approx 1}{\approx} \omega_0 + c_2\left(p_k^\star - p_k\right) \quad (p-\omega \text{ droop})$$

$$\frac{d}{dt}\|v_k\|$$

Properties of virtual oscillator control

1. desired target dynamics can be realized via **fully decentralized control**

$$\frac{d}{dt}v_k = \underbrace{\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} v_k}_{\text{rotation at } \omega_0} + \underbrace{c_1 \cdot \left(v^{\star 2} - \|v_k\|^2\right) v_k}_{\text{local amplitude regulation}} + \underbrace{c_2 \cdot R(\kappa) \left(\frac{1}{v^{\star 2}} \begin{bmatrix} q_k^\star & p_k^\star \\ -p_k^\star & q_k^\star \end{bmatrix} v_k - i_{o,k} \right)}_{\text{synchronization through physics}}$$

2. connection to **droop control** revealed in polar coordinates (for inductive grid)

$$\frac{d}{dt}\theta_k = \omega_0 + c_1 \left(\frac{p_k^\star}{v^{\star 2}} - \frac{p_k}{\|v_k\|^2} \right) \underset{\|v_k\| \approx 1}{\approx} \omega_0 + c_2 \left(p_k^\star - p_k\right) \quad (p - \omega \text{ droop})$$

$$\frac{d}{dt}\|v_k\| \underset{\|v_k\| \approx 1}{\approx} c_2 \left(q_k^\star - q_k\right) + c_1 \left(v^\star - \|v_k\|\right) \quad (q - \|v\| \text{ droop})$$

Properties of virtual oscillator control

1. desired target dynamics can be realized via **fully decentralized control**

$$\frac{d}{dt}v_k = \underbrace{\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} v_k}_{\text{rotation at } \omega_0} + c_1 \cdot \underbrace{\left(v^{\star 2} - \|v_k\|^2\right) v_k}_{\text{local amplitude regulation}} + c_2 \cdot \underbrace{R(\kappa)\left(\frac{1}{v^{\star 2}} \begin{bmatrix} q_k^\star & p_k^\star \\ -p_k^\star & q_k^\star \end{bmatrix} v_k - i_{o,k}\right)}_{\text{synchronization through physics}}$$

2. connection to **droop control** revealed in polar coordinates (for inductive grid)

$$\frac{d}{dt}\theta_k = \omega_0 + c_1\left(\frac{p_k^\star}{v^{\star 2}} - \frac{p_k}{\|v_k\|^2}\right) \underset{\|v_k\|\approx 1}{\approx} \omega_0 + c_2\left(p_k^\star - p_k\right) \quad (p-\omega \text{ droop})$$

$$\frac{d}{dt}\|v_k\| \underset{\|v_k\|\approx 1}{\approx} c_2\left(q_k^\star - q_k\right) + c_1\left(v^\star - \|v_k\|\right) \quad (q-\|v\| \text{ droop})$$

3. **almost global asymptotic stability** if

Properties of virtual oscillator control

1. desired target dynamics can be realized via **fully decentralized control**

$$\frac{d}{dt}v_k = \underbrace{\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} v_k}_{\text{rotation at } \omega_0} + c_1 \cdot \underbrace{\left(v^{\star 2} - \|v_k\|^2\right) v_k}_{\text{local amplitude regulation}} + c_2 \cdot \underbrace{R(\kappa)\left(\frac{1}{v^{\star 2}} \begin{bmatrix} q_k^\star & p_k^\star \\ -p_k^\star & q_k^\star \end{bmatrix} v_k - i_{o,k}\right)}_{\text{synchronization through physics}}$$

2. connection to **droop control** revealed in polar coordinates (for inductive grid)

$$\frac{d}{dt}\theta_k = \omega_0 + c_1\left(\frac{p_k^\star}{v^{\star 2}} - \frac{p_k}{\|v_k\|^2}\right) \underset{\|v_k\|\approx 1}{\approx} \omega_0 + c_2\left(p_k^\star - p_k\right) \quad (p-\omega \text{ droop})$$

$$\frac{d}{dt}\|v_k\| \underset{\|v_k\|\approx 1}{\approx} c_2\left(q_k^\star - q_k\right) + c_1\left(v^\star - \|v_k\|\right) \quad (q-\|v\| \text{ droop})$$

3. **almost global asymptotic stability** if

- **power transfer** “small” compared to **network connectivity**
- **amplitude control** “slower” than **synchronization control**

Details on stability condition

If $\boxed{\ldots}$ condition holds, the system is **almost globally asymptotically stable** with respect to a **limit cycle** corresponding to a **pre-specified** solution of the **AC power-flow** equations at a **synchronous** frequency ω_0.

Details on stability condition

If $\boxed{\ldots}$ condition holds, the system is **almost globally asymptotically stable** with respect to a **limit cycle** corresponding to a **pre-specified** solution of the **AC power-flow** equations at a **synchronous** frequency ω_0.

e.g., for resistive grid:

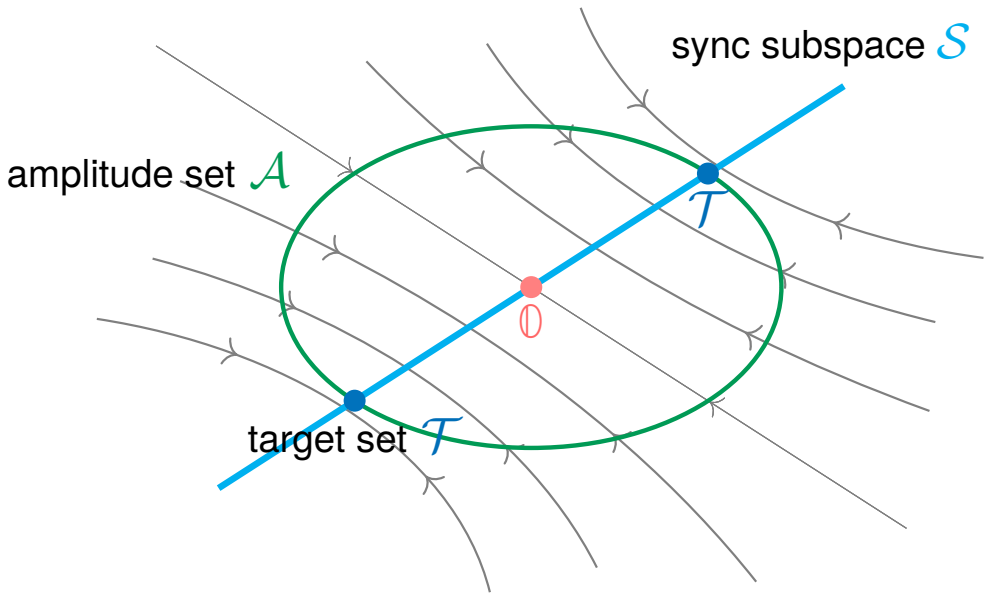
$$\frac{1}{2} \underbrace{\lambda_2}_{\text{algebraic connectivity}} > \max_k \sum_{j=1}^{n} \frac{1}{v^{\star 2}} \underbrace{|p_{jk}|}_{\text{power transfer}} + \frac{c_1}{c_2} v^\star$$

- **power transfer** p_{jk} "small" compared to **network connectivity** λ_2
- **amplitude control** "slower" than **synchronization control**: $c_1/c_2 \ll 1$

Proof sketch: Lyapunov & center manifold

Lyapunov function: $V(v) = \frac{1}{2}\text{dist}(v, \mathcal{S})^2 + \frac{c_2}{v^{\star 2}} \sum_k \left(v^{\star 2} - \|v_k\|^2\right)^2$

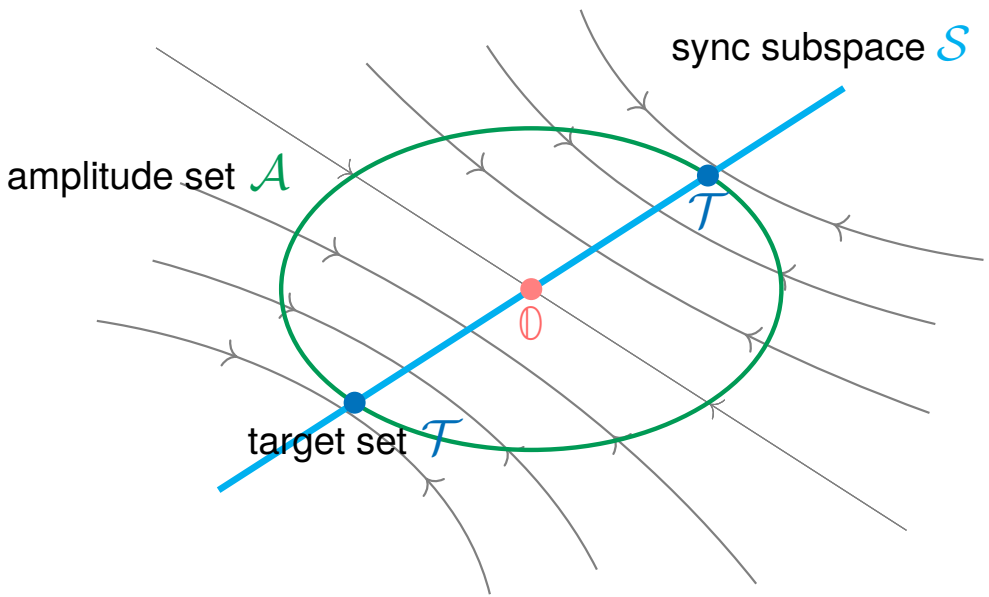
(decreasing under parametric condition)



Proof sketch: Lyapunov & center manifold

Lyapunov function: $V(v) = \frac{1}{2}\mathrm{dist}(v, \mathcal{S})^2 + \frac{c_2}{v^{\star 2}} \sum_k \left(v^{\star 2} - \|v_k\|^2\right)^2$

(decreasing under parametric condition)



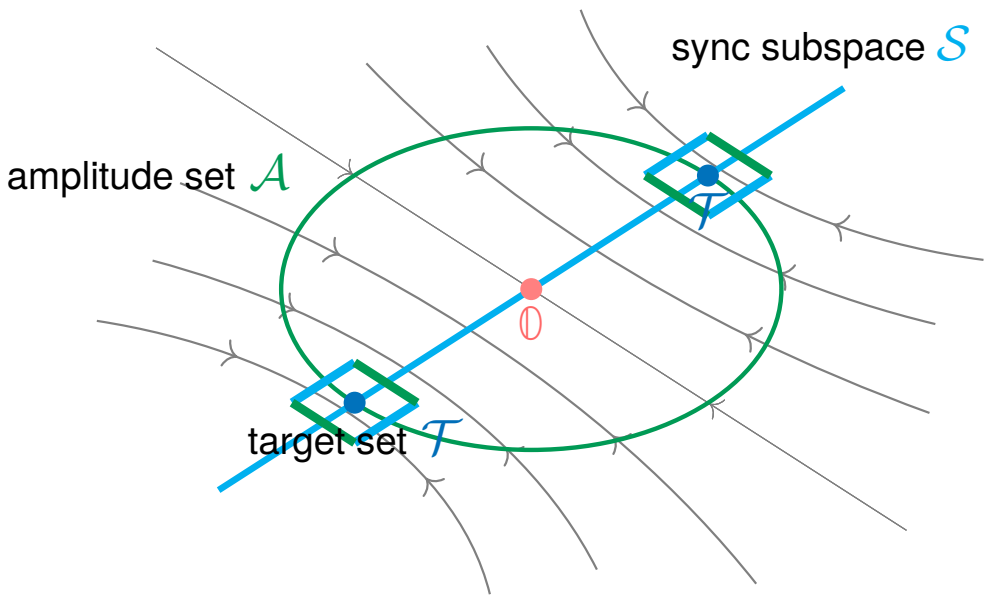
$\mathcal{T} \cup \mathbb{0}$ is globally attractive

$$\lim_{t\to\infty} \mathrm{dist}\big(v(t), \mathcal{T} \cup \mathbb{0}\big) = 0$$

Proof sketch: Lyapunov & center manifold

Lyapunov function: $V(v) = \frac{1}{2}\text{dist}(v, \mathcal{S})^2 + \frac{c_2}{v^{\star 2}} \sum_k \left(v^{\star 2} - \|v_k\|^2\right)^2$

(decreasing under parametric condition)



$\mathcal{T} \cup \mathbb{0}$ is globally attractive

$$\lim_{t\to\infty} \text{dist}\big(v(t), \mathcal{T} \cup \mathbb{0}\big) = 0$$

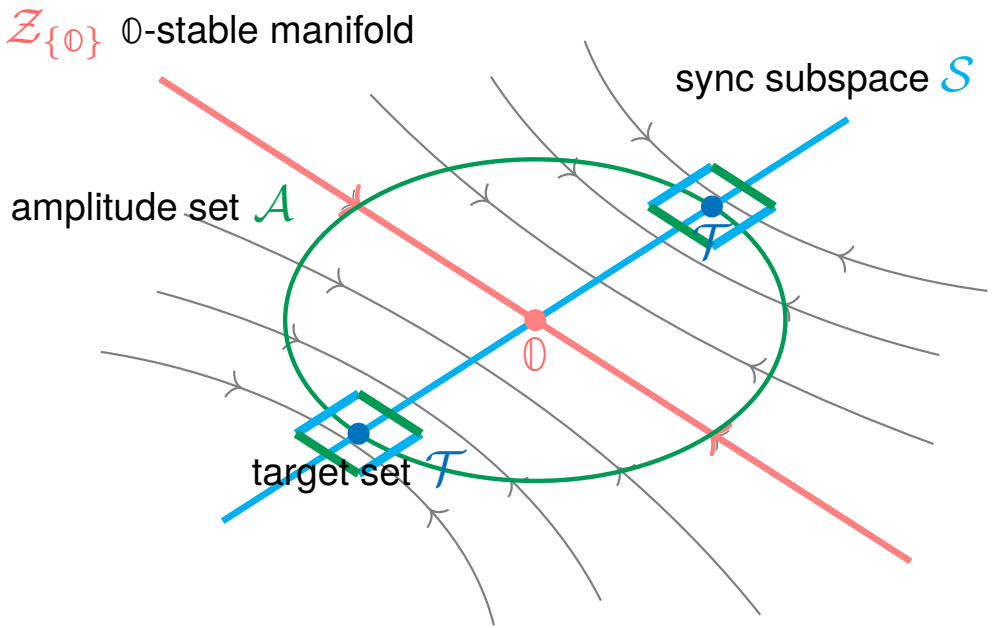
$\mathcal{T}$ is stable

$$\text{dist}\big(v(t), \mathcal{T}\big) \leq \mathcal{K} \circ \text{dist}\big(v_0, \mathcal{T}\big)$$

Proof sketch: Lyapunov & center manifold

Lyapunov function: $V(v) = \frac{1}{2}\text{dist}(v, \mathcal{S})^2 + \frac{c_2}{v^{\star 2}} \sum_k \left(v^{\star 2} - \|v_k\|^2\right)^2$

(decreasing under parametric condition)



$\mathcal{T} \cup \mathbb{0}$ is globally attractive

$$\lim_{t\to\infty} \text{dist}\big(v(t), \mathcal{T} \cup \mathbb{0}\big) = 0$$

$\mathcal{T}$ is stable

$$\text{dist}\big(v(t), \mathcal{T}\big) \leq \mathcal{K} \circ \text{dist}\big(v_0, \mathcal{T}\big)$$

$\mathcal{T}$ is almost globally attractive

$\mathbb{0}$ exponentially unstable

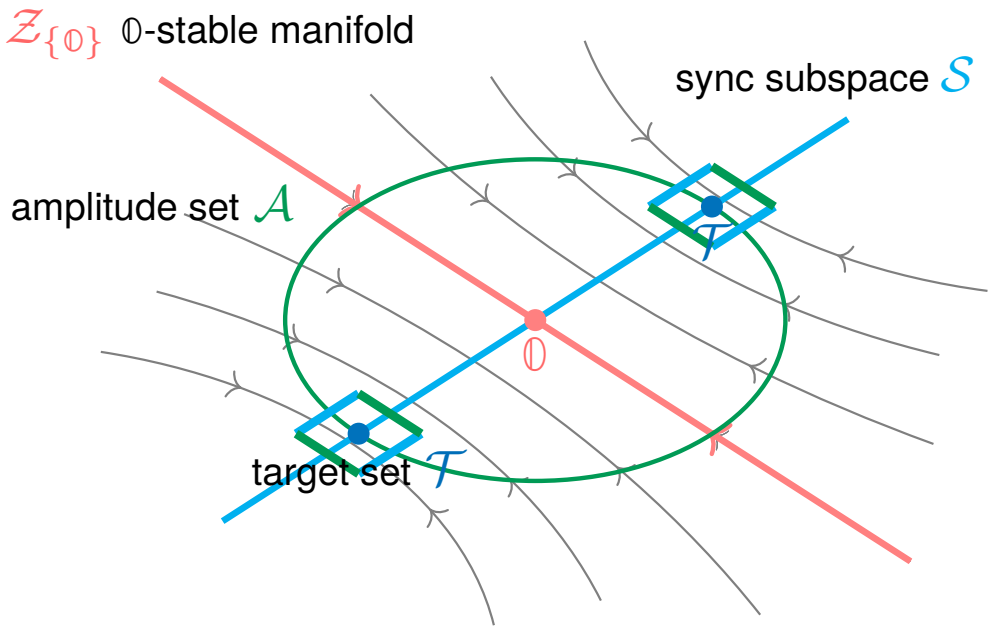
$\implies$ $\mathcal{Z}_{\{\mathbb{0}\}}$ has measure zero

$$\forall v_0 \notin \mathcal{Z}_{\{\mathbb{0}\}} : \lim_{t\to\infty} \text{dist}\big(v(t), \mathcal{T}\big) = 0$$

Proof sketch: Lyapunov & center manifold

Lyapunov function: $V(v) = \frac{1}{2}\text{dist}(v, \mathcal{S})^2 + \frac{c_2}{v^{\star 2}} \sum_k \left(v^{\star 2} - \|v_k\|^2\right)^2$

(decreasing under parametric condition)



$\mathcal{T} \cup \mathbb{0}$ is globally attractive

$$\lim_{t\to\infty} \text{dist}\big(v(t), \mathcal{T} \cup \mathbb{0}\big) = 0$$

$\mathcal{T}$ is stable

$$\text{dist}\big(v(t), \mathcal{T}\big) \leq \mathcal{K} \circ \text{dist}\big(v_0, \mathcal{T}\big)$$

$\mathcal{T}$ is almost globally attractive

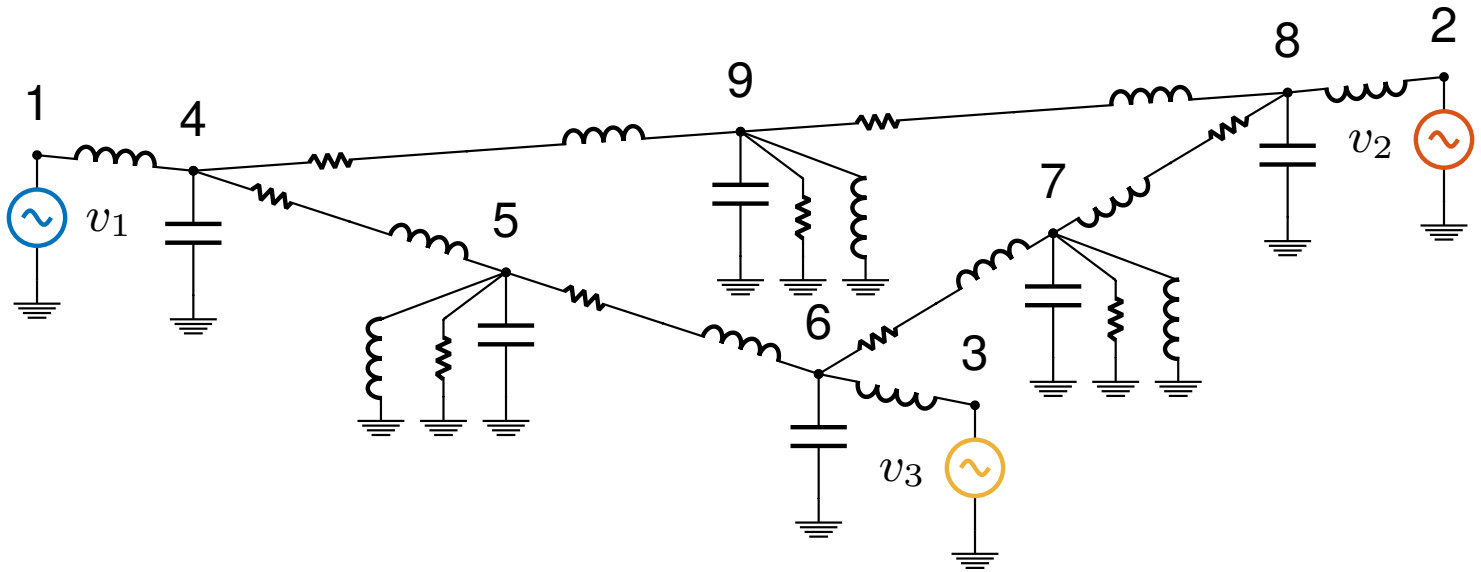
$\mathbb{0}$ exponentially unstable

$\implies$ $\mathcal{Z}_{\{\mathbb{0}\}}$ has measure zero

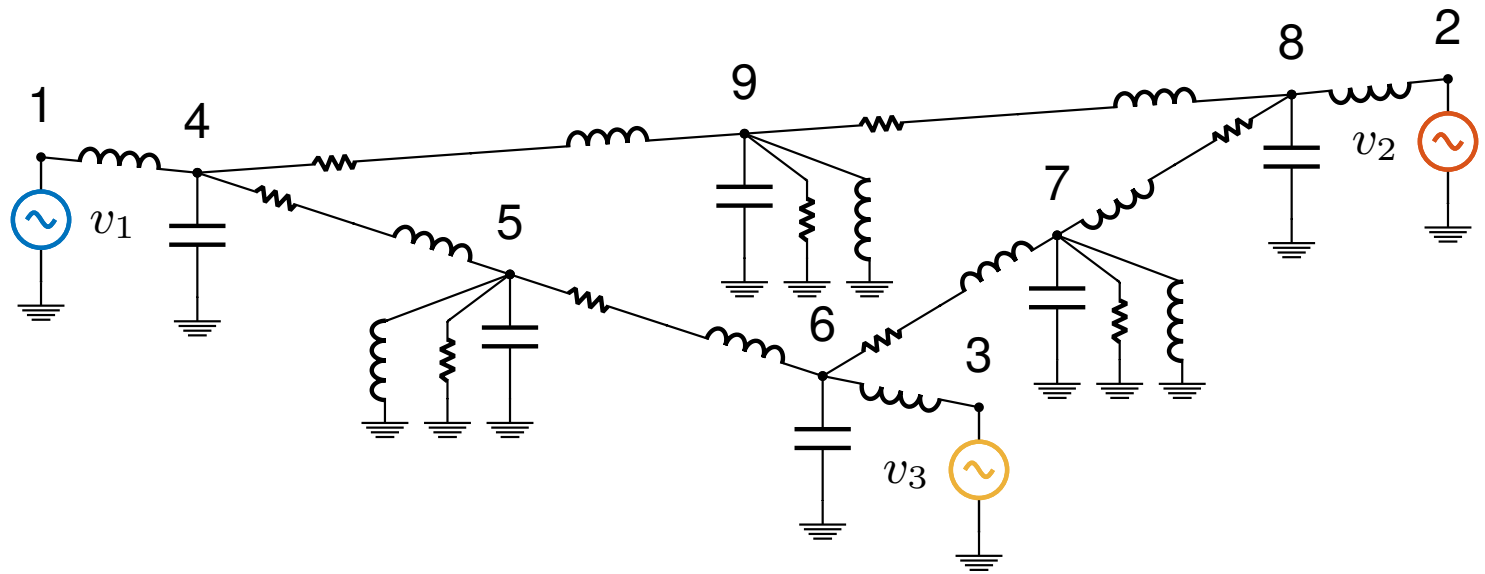
$$\forall v_0 \notin \mathcal{Z}_{\{\mathbb{0}\}} : \lim_{t\to\infty} \text{dist}\big(v(t), \mathcal{T}\big) = 0$$

stability & almost global attractivity $\implies$ **almost global asymptotic stability**

Case study: IEEE 9 Bus system



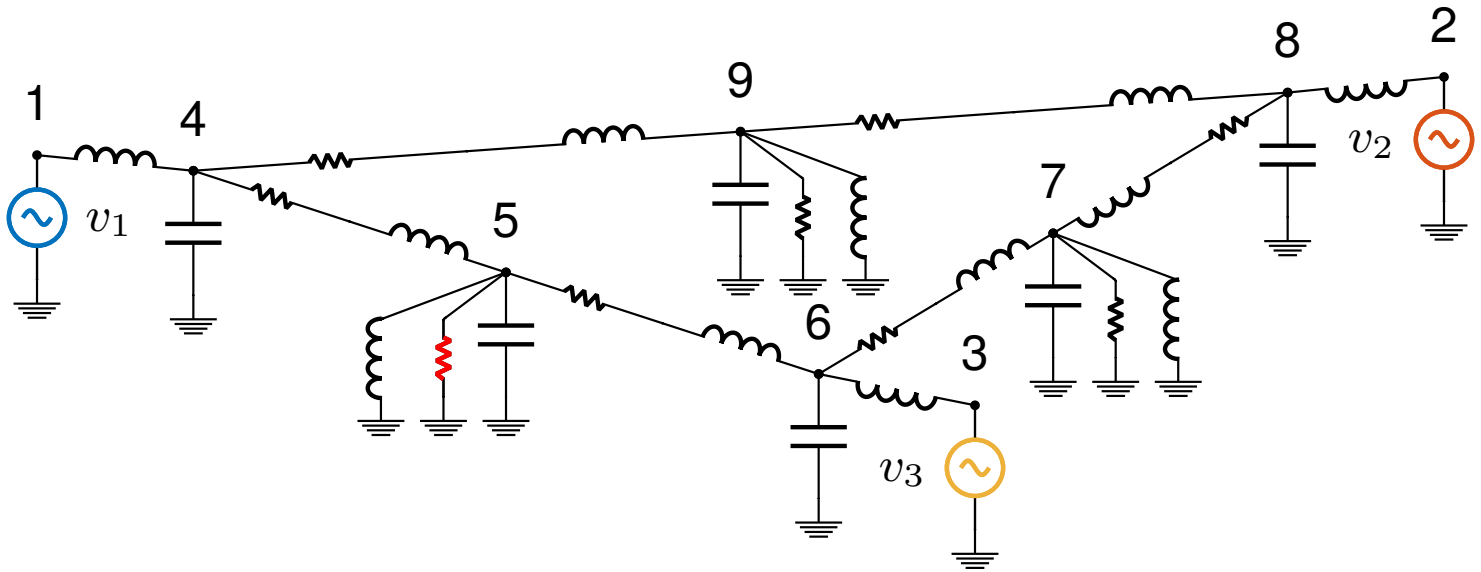
Case study: IEEE 9 Bus system



t = 0 s: black start of three inverters

- **initial state:** $\|v_k(0)\| \approx 10^{-3}$
- **convergence** to **set-point**

Case study: IEEE 9 Bus system



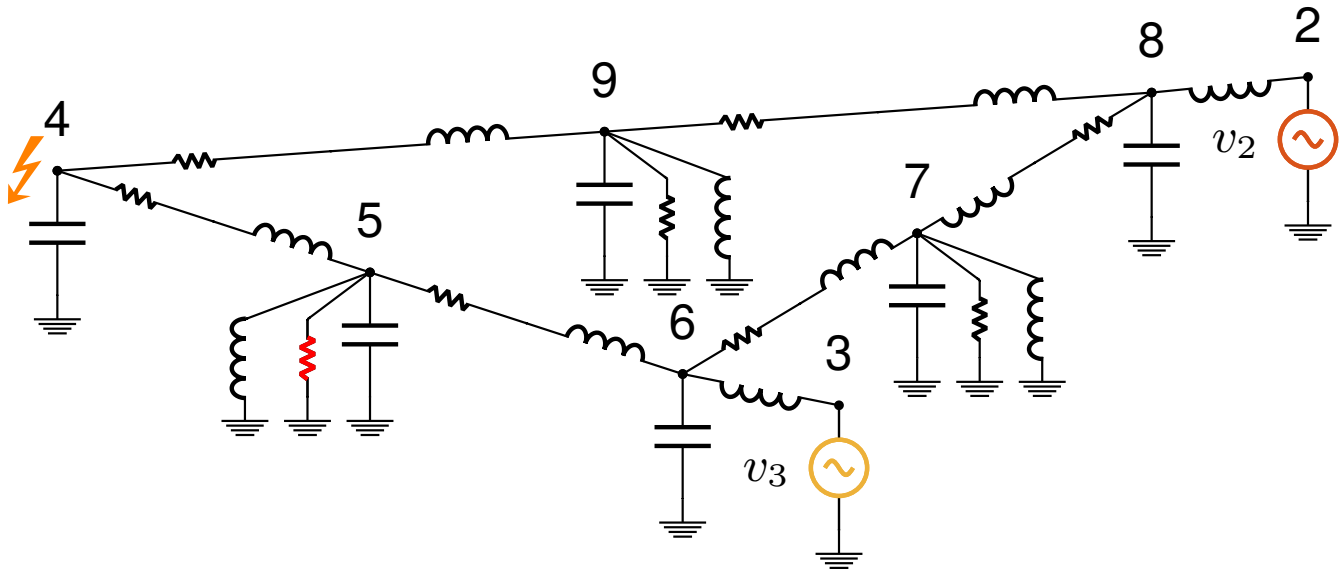
t = 0 s: black start of three inverters

- **initial state:** $\|v_k(0)\| \approx 10^{-3}$
- **convergence** to **set-point**

t = 5 s: load step-up

- 20% **load increase** at bus 5
- consistent **power sharing**

Case study: IEEE 9 Bus system



t = 0 s: black start of three inverters

- **initial state:** $\|v_k(0)\| \approx 10^{-3}$
- **convergence** to **set-point**

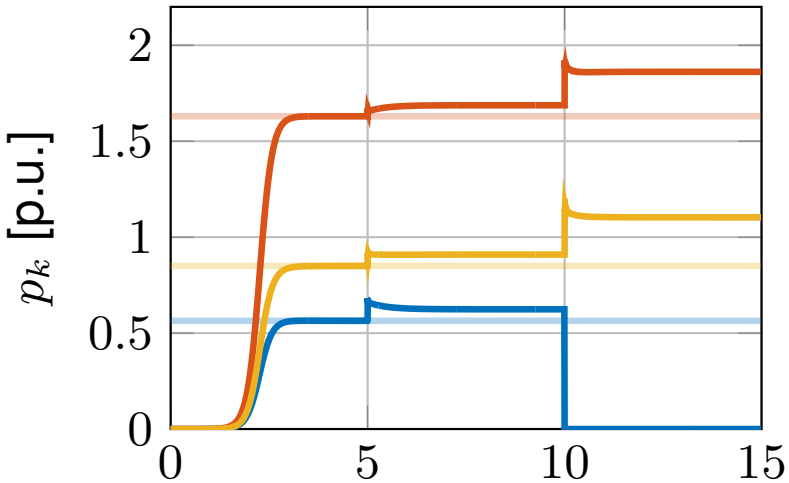
t = 5 s: load step-up

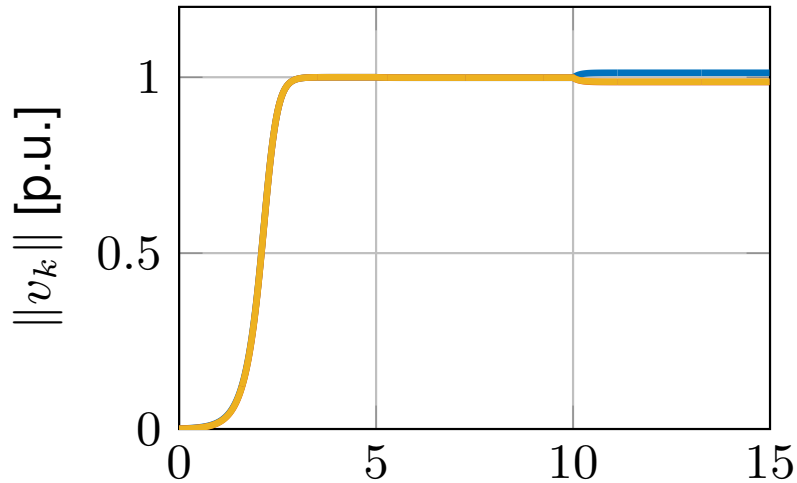
- 20% **load increase** at bus 5
- consistent **power sharing**

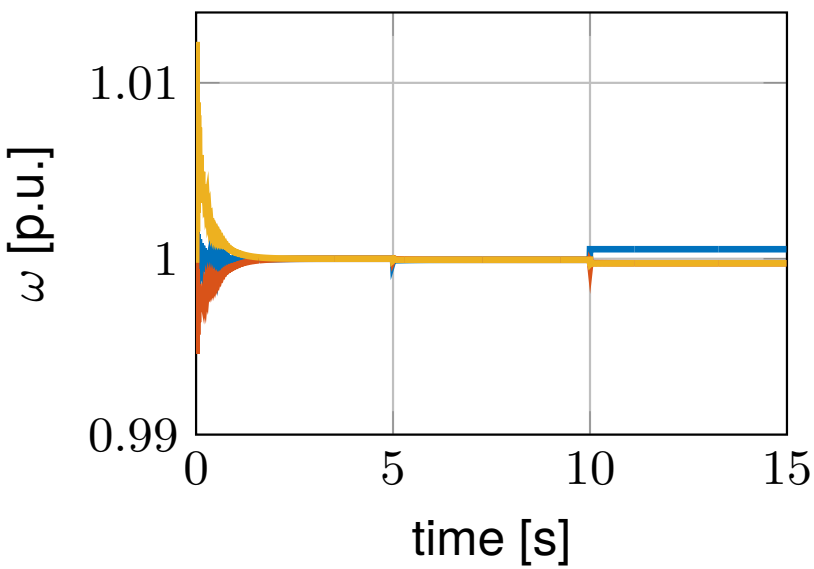
t = 10 s: loss of inverter 1

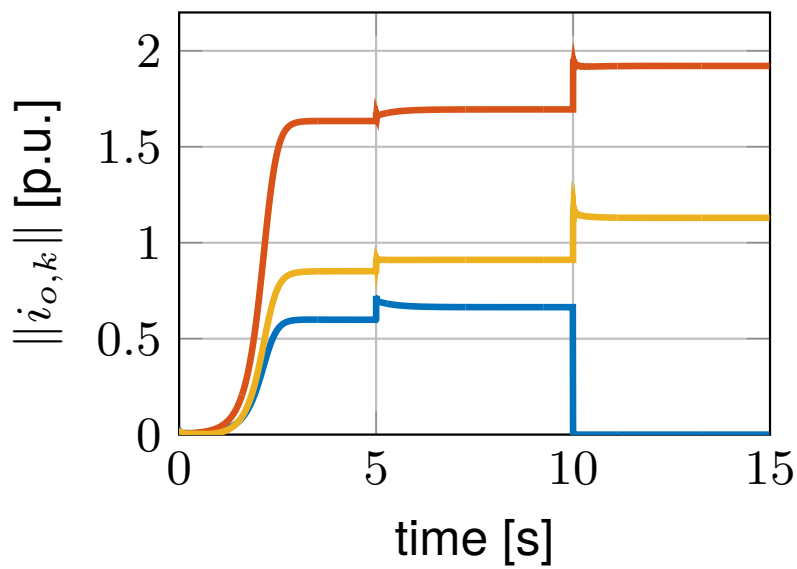
- the **remaining** inverters **synchronize**
- they **supply** the load **sharing power**

Simulation of IEEE 9 Bus system





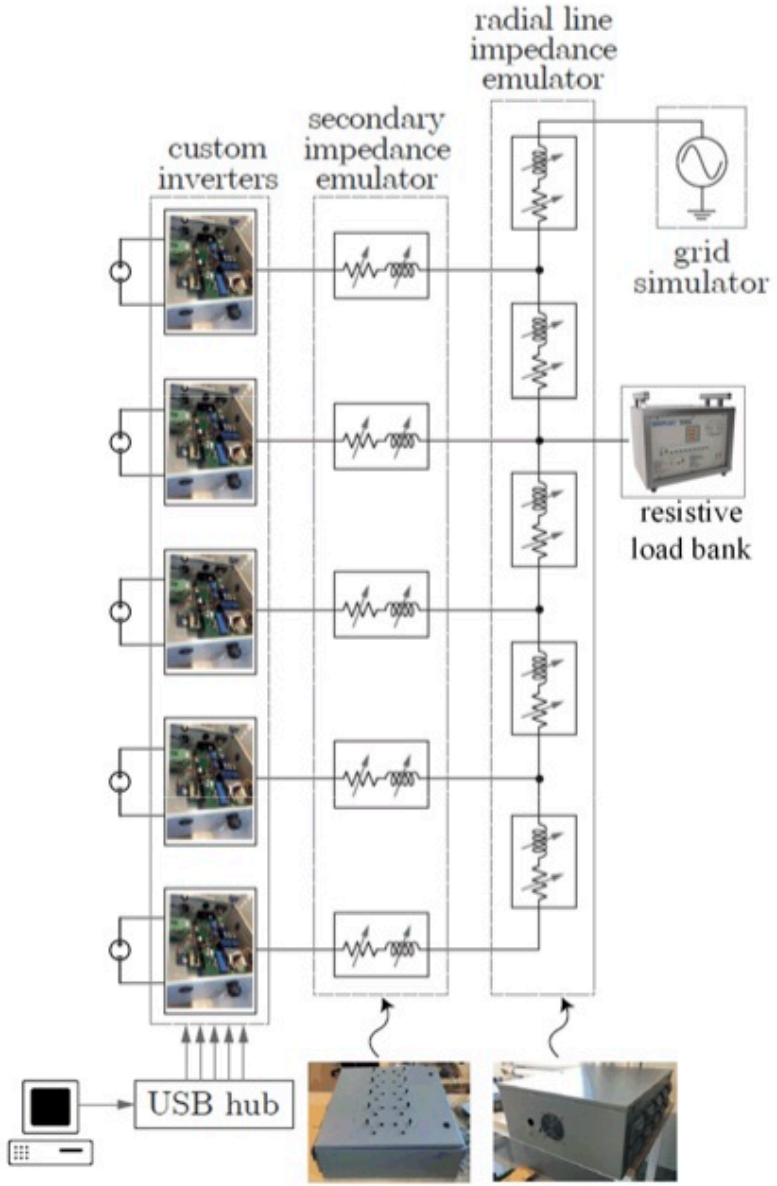




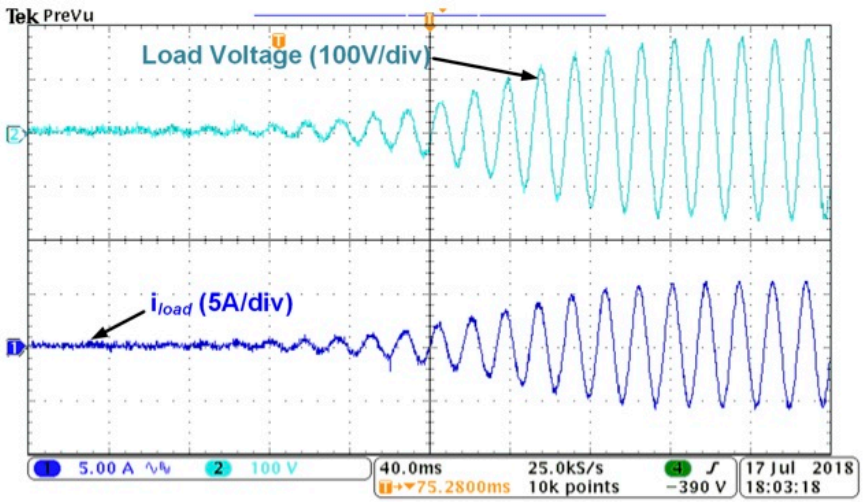
Experimental setup @ NREL



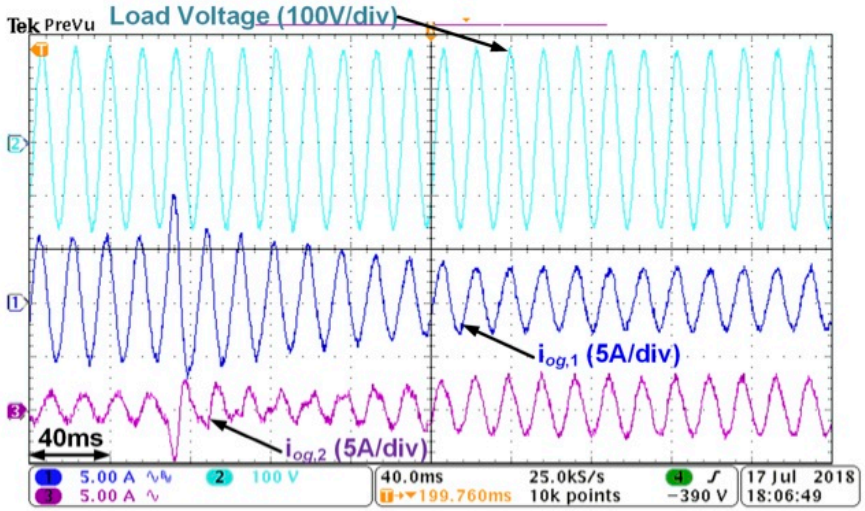




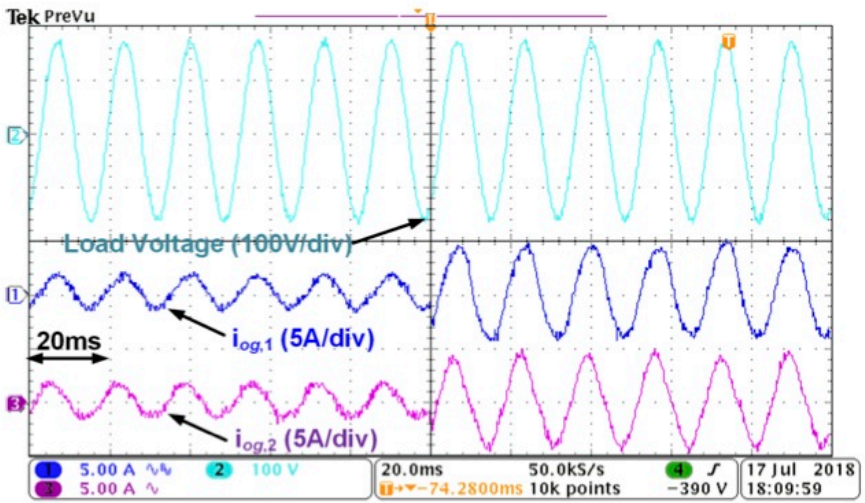
Experimental results



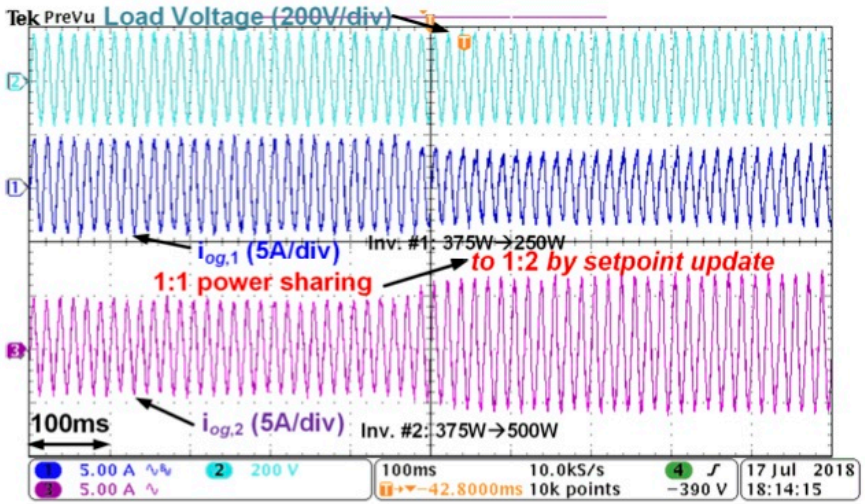
black start of inverter #1 under 500 W load (making use of almost global stability)



connecting inverter #2 while inverter #1 is regulating the grid under 500 W load



250 W to 750 W load transient with two inverters active



change of setpoint: $p^\star$ of inverter #2 updated from 250 W to 500 W

Details & initial condition for further reading

Global Phase and Magnitude Synchronization of Coupled Oscillators With Application to the Control of Grid-Forming Power Inverters

Marcello Colombino, Dominic Groß, *Member, IEEE*, Jean-Sébastien Brouillon, and Florian Dörfler, *Member, IEEE*

***Abstract*—In this paper, we explore a new approach to synchronization of coupled oscillators. In contrast to the celebrated Kuramoto model, we do not work in polar coordinates and do not consider oscillations of fixed magnitude. We propose a synchronizing feedback based on relative state information and local measurements that induces consensus-like dynamics. We show that, under a mild stability condition, the combination of the synchronizing feedback with a decentralized magnitude control law renders the oscillators' almost globally asymptotically stable with respect to set points for the phase shift, frequency, and magnitude. We apply these result to rigorously solve an**

distribution grid may operate without conventional bulk generation by synchronous machines. In either case, the power grid faces great challenges due to the loss of the machines' rotational inertia and the loss of self-synchronization dynamics inherent to synchronous machines.

The prevalent approaches to controlling inverters in the future grid are based on mimicking the physical characteristics and controls of synchronous machines [1]–[3]. On the one hand, this approach is appealing because it results in a well-studied closed-loop behavior compatible with the legacy power system.

The Effect of Transmission-Line Dynamics on Grid-Forming Dispatchable Virtual Oscillator Control

Dominic Groß, *Member, IEEE*, Marcello Colombino, Jean-Sébastien Brouillon, and Florian Dörfler, *Member, IEEE*

***Abstract*—In this paper, we analyze the effect of transmission line dynamics on grid-forming control for inverter-based ac power systems. In particular, we investigate a dispatchable virtual oscillator control (dVOC) strategy that was recently proposed in the literature. When the dynamics of the transmission lines are neglected, that is, if an algebraic model of the transmission network is used, dVOC ensures almost global asymptotic stability of a network of ac power inverters with respect to a prespecified solution of the ac power-flow equations. While this approximation is typically justified for conventional power systems, the electromagnetic transients of the transmission lines can com-**

inverter is not limited to power tracking, but acts as a controlled voltage source that can change its power output (thanks to storage or curtailment), and is controlled to contribute to the stability of the grid. Most of the common approaches of grid-forming control focus on droop control [1], [2]. Other popular approaches are based on mimicking the physical characteristics and controls of synchronous machines [3], [4] or controlling inverters to behave like virtual Liénard-type oscillators [5]–[7]. While strategies based on machine-emulation are compatible with the legacy power system, they use a system (the inverter)

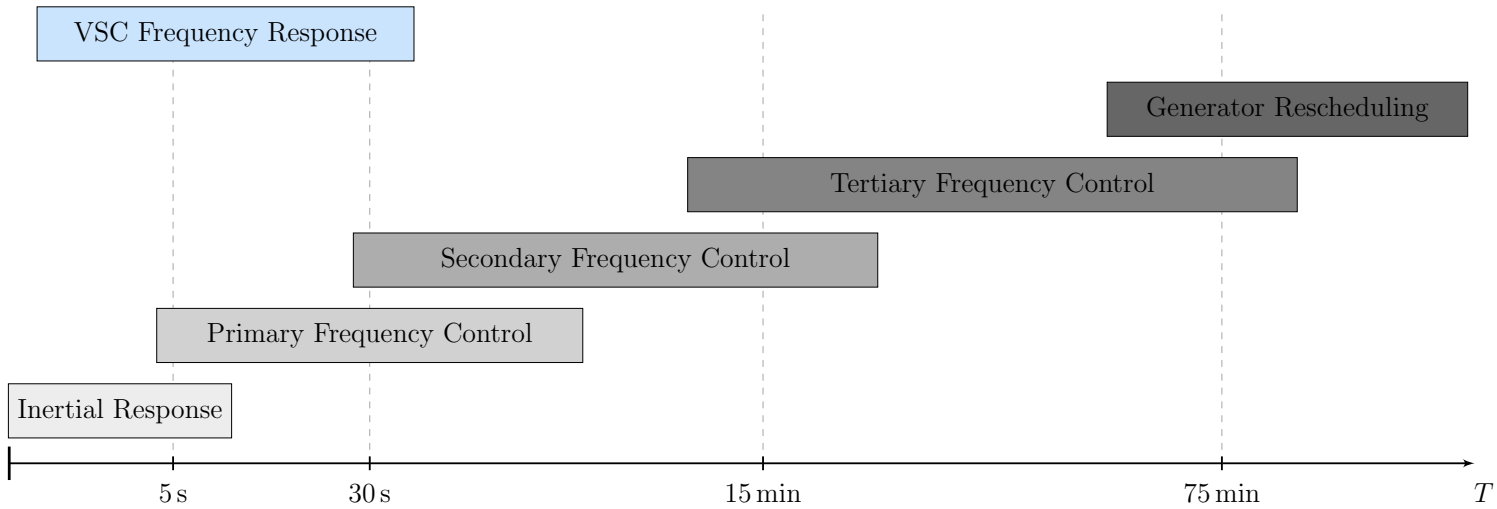
Outline

- Virtual Synchronous Machine Emulation
- Droop Control
- Matching Control
- Virtual Oscillator Control (VOC)
- Time-Scale Separation Issues
- Comparison

Time-scale separations (?) – old and new

figure provided by Uroš Marković

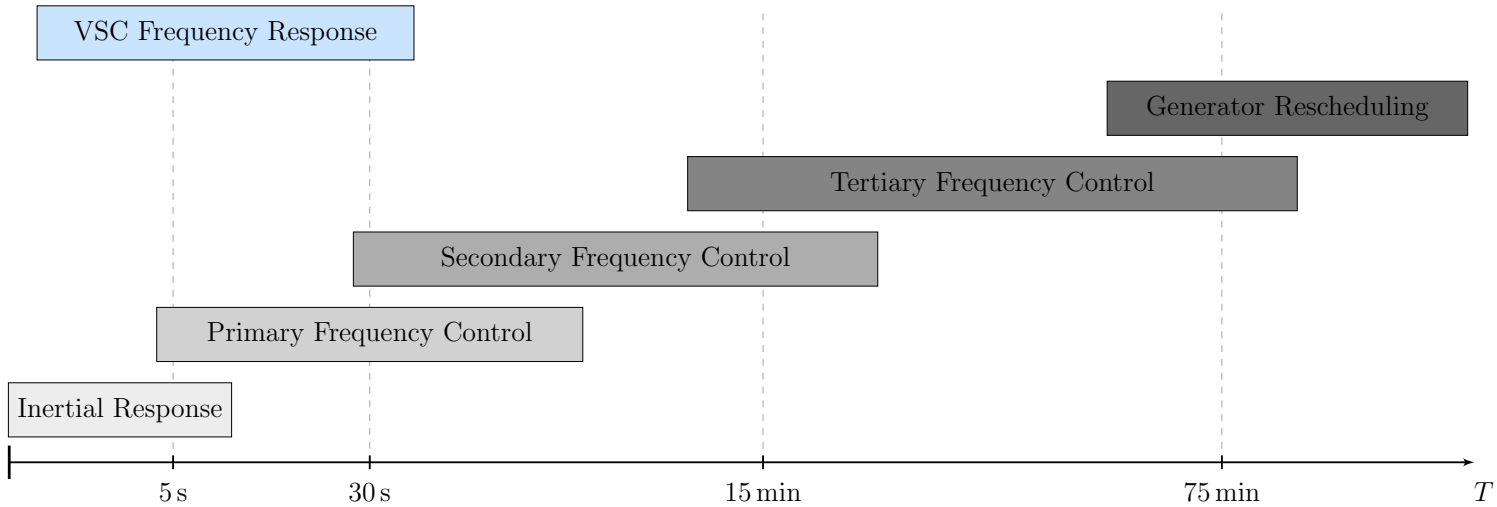
power system operational time scales



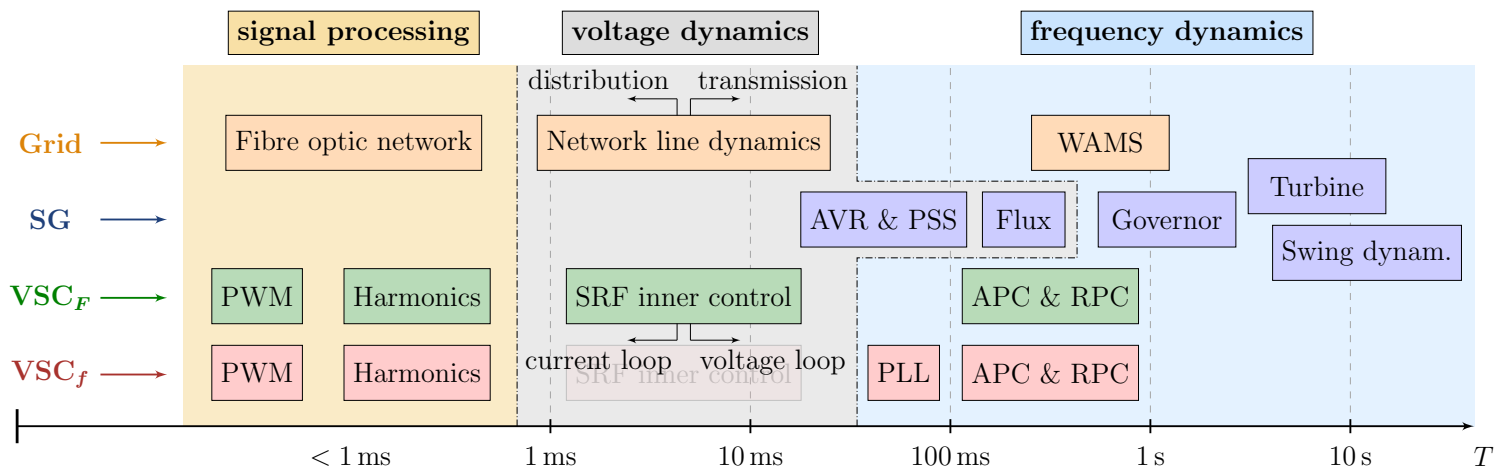
Time-scale separations (?) – old and new

figure provided by Uroš Marković

power system operational time scales



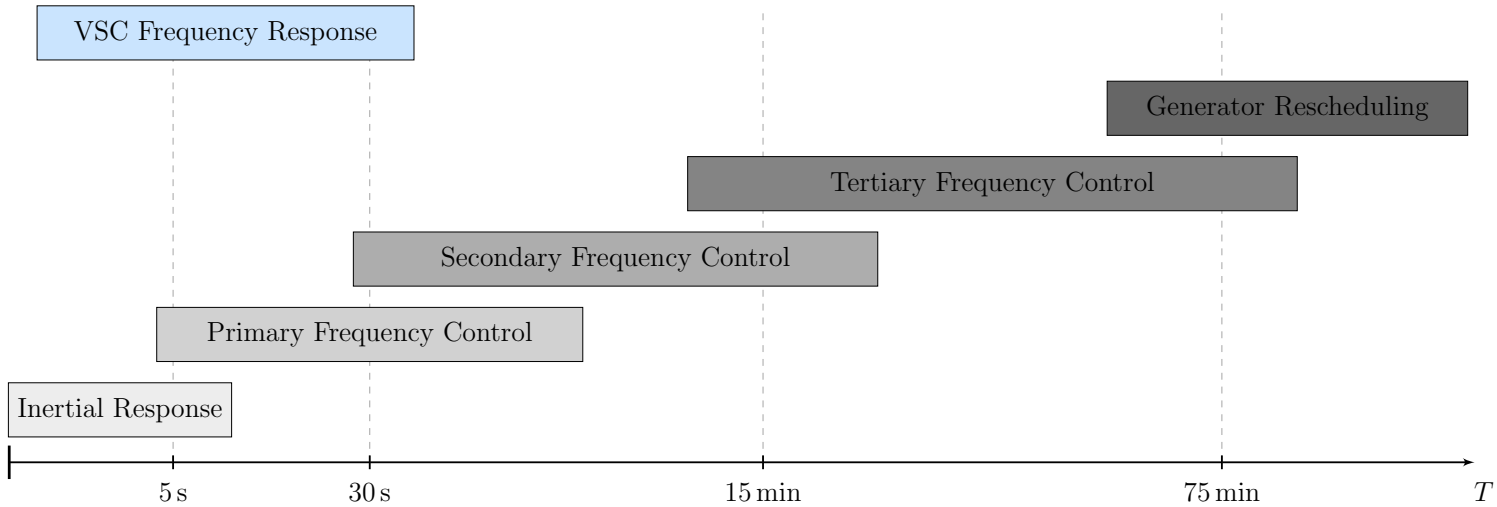
fast time scales: converter/generator controls & physics



Time-scale separations (?) – old and new

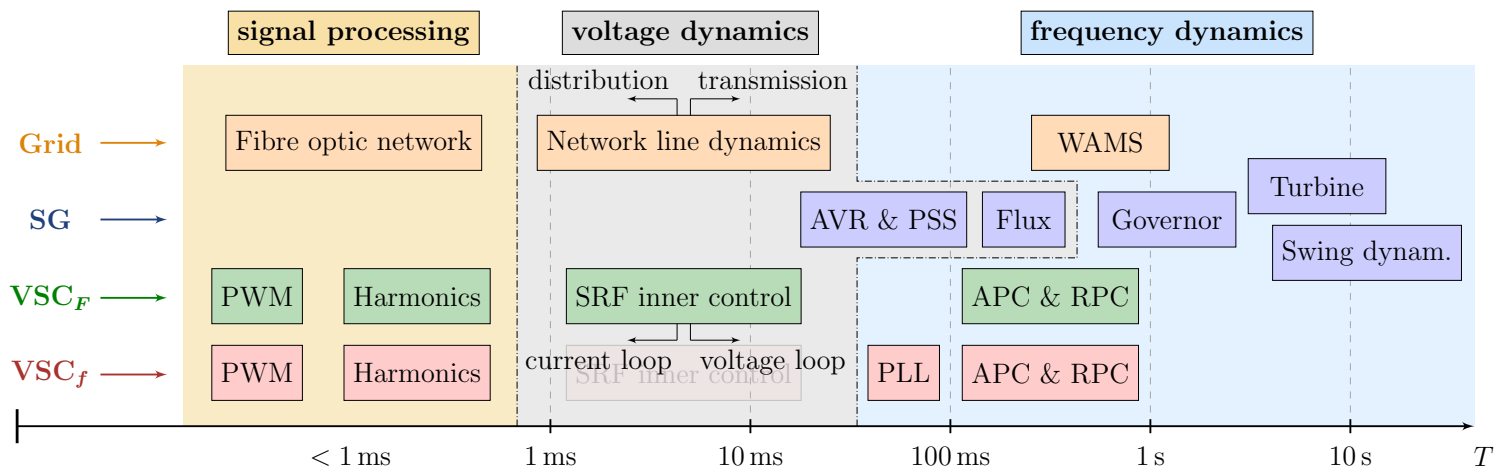
figure provided by Uroš Marković

power system operational time scales



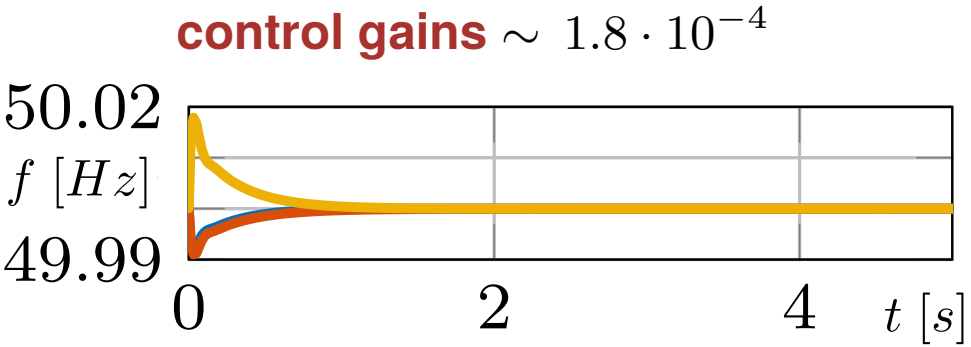
fast time scales: converter/generator controls & physics

→ separated aside from **line dynamics**



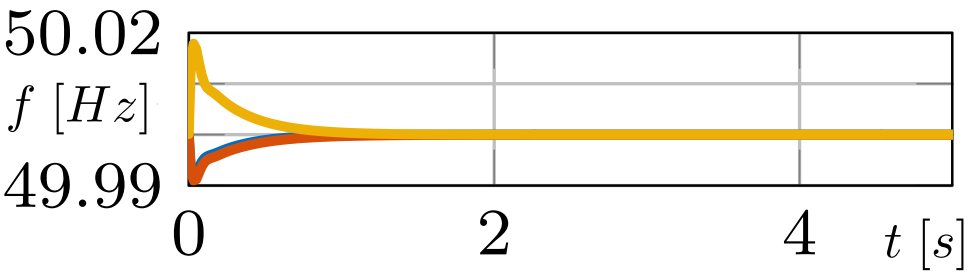
the following *issues* apply to all grid-forming control strategies, but we discuss them here only for VOC

Dropping assumptions: dynamic lines

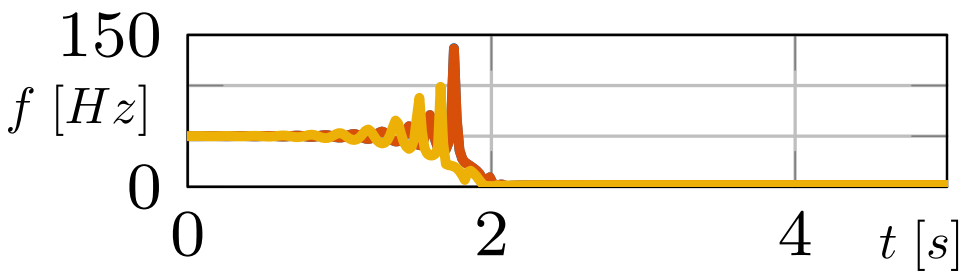


Dropping assumptions: dynamic lines

control gains $\sim 1.8 \cdot 10^{-4}$

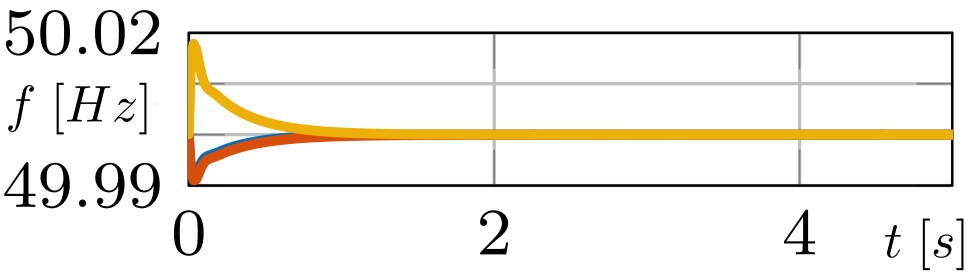


control gains $\sim 1.8 \cdot 10^{-3}$

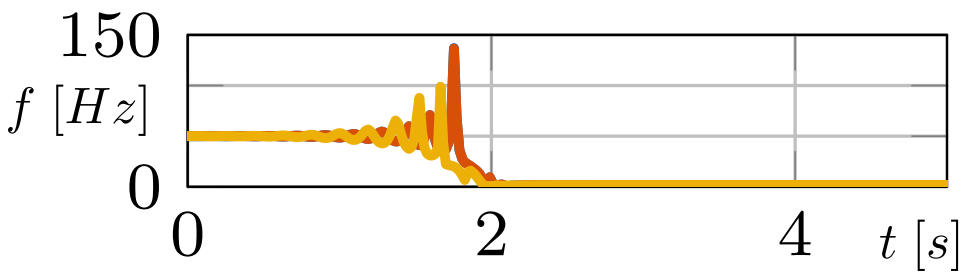


Dropping assumptions: dynamic lines

control gains $\sim 1.8 \cdot 10^{-4}$



control gains $\sim 1.8 \cdot 10^{-3}$



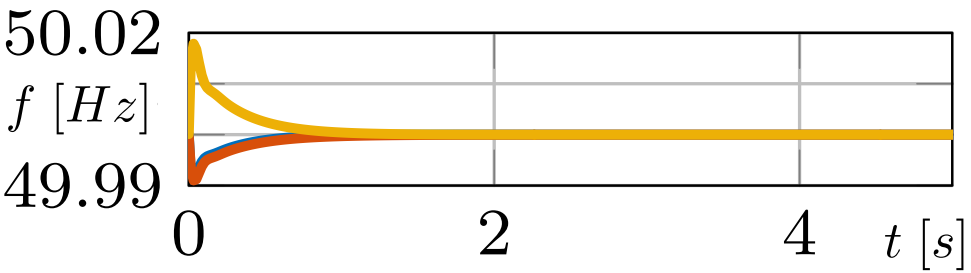
observations

- inverter control **interferes** with the line dynamics
- controller needs to be artificially **slowed down**
- well recognized problem

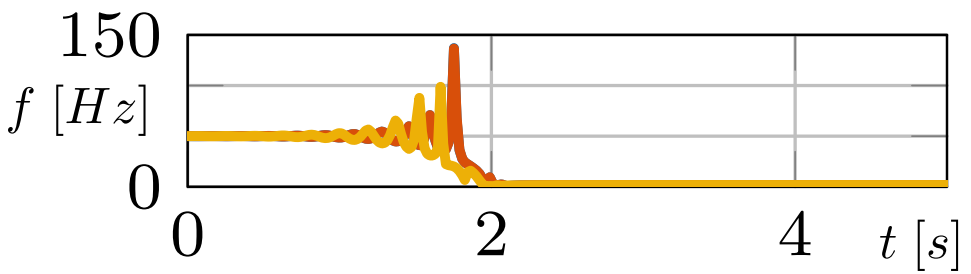
[Vorobev, Huang, Hosaini, & Turitsyn,'17]

Dropping assumptions: dynamic lines

control gains $\sim 1.8 \cdot 10^{-4}$



control gains $\sim 1.8 \cdot 10^{-3}$



observations

- inverter control **interferes** with the line dynamics
- controller needs to be artificially **slowed down**
- well recognized problem

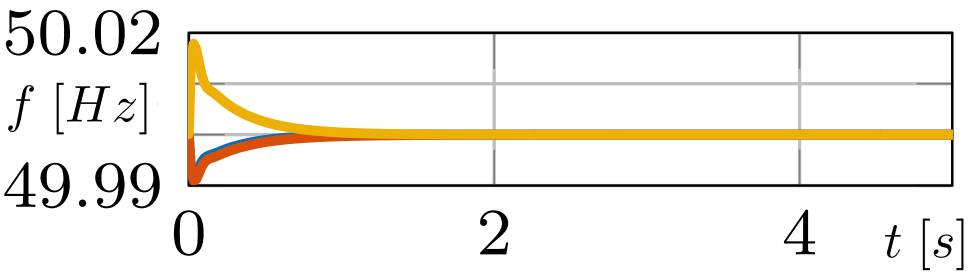
[Vorobev, Huang, Hosaini, & Turitsyn,'17]

networked control reason

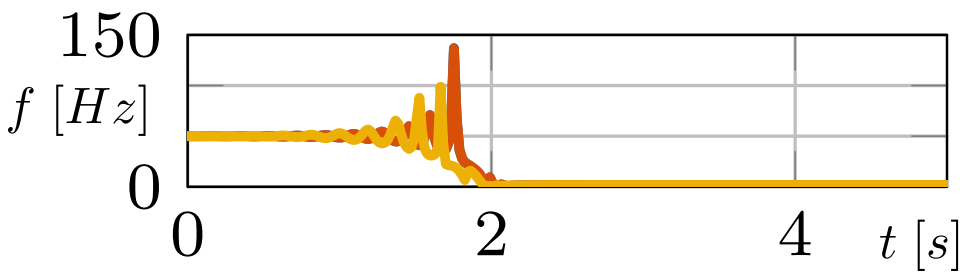
- **communication through currents** to infer voltages
- very inductive lines **delay** the information transfer
- the controller must be **slow** in very inductive networks

Dropping assumptions: dynamic lines

control gains $\sim 1.8 \cdot 10^{-4}$



control gains $\sim 1.8 \cdot 10^{-3}$



re-do the math leading to new condition:

amplitude control slower than **sync control** **slower than line dynamics**

observations

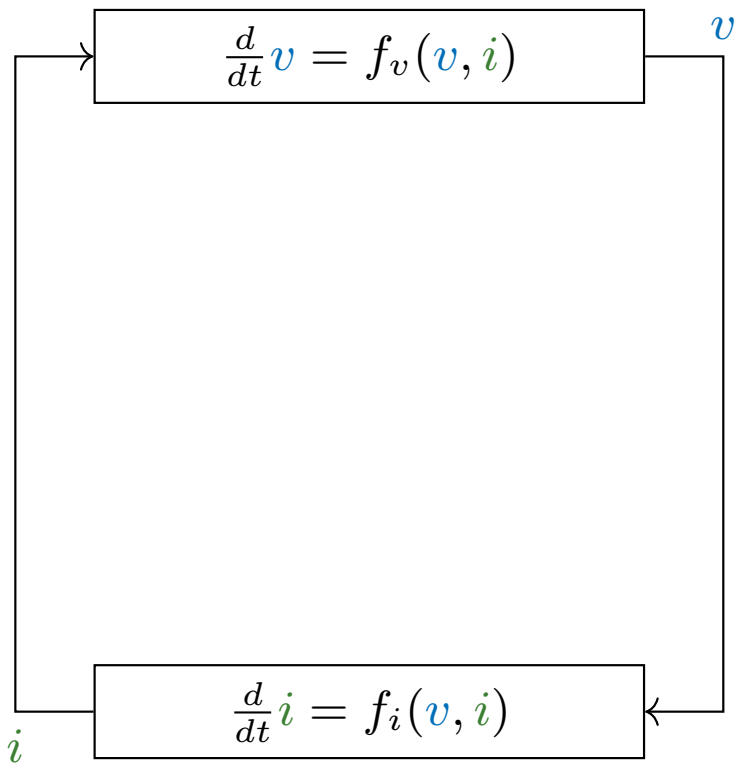
- inverter control **interferes** with the line dynamics
- controller needs to be artificially **slowed down**
- well recognized problem

[Vorobev, Huang, Hosaini, & Turitsyn,'17]

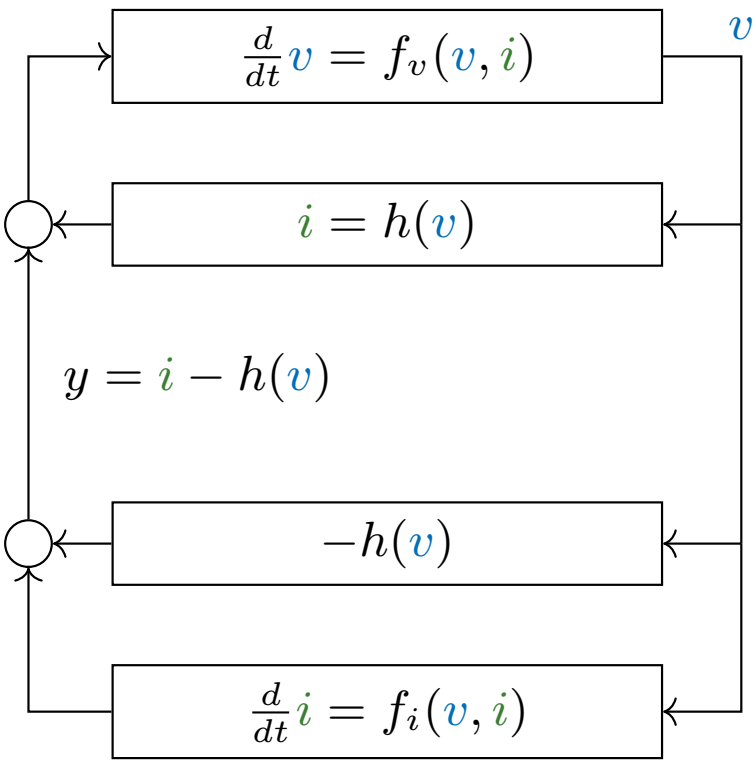
networked control reason

- **communication through currents** to infer voltages
- very inductive lines **delay** the information transfer
- the controller must be **slow** in very inductive networks

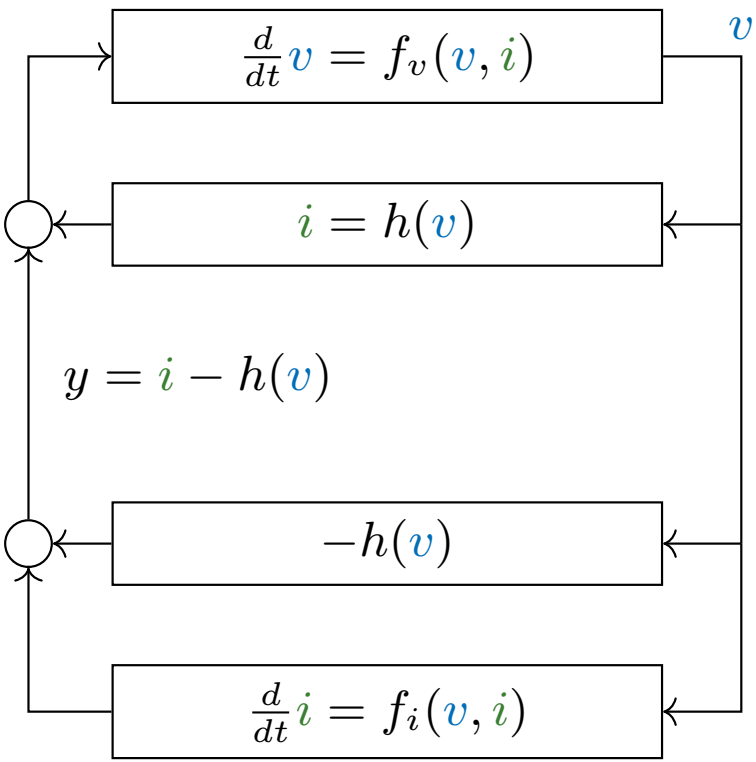
Proof extension: perturbation-inspired Lyapunov



Proof extension: perturbation-inspired Lyapunov



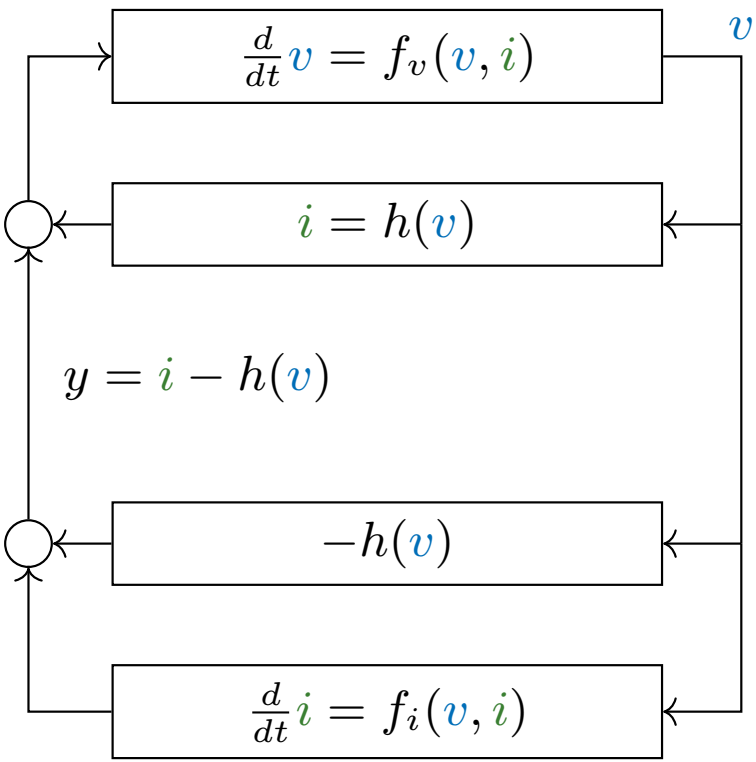
Proof extension: perturbation-inspired Lyapunov



Individual Lyapunov functions

- slow system: $V(v)$ for $\frac{d}{dt}v = f_v(v, h(v))$
- fast system: $W(y)$ for $\frac{d}{dt}y = f_i(v, y + h(v))$
  where $\frac{d}{dt}v = 0$ & coordinate $y = i - h(v)$

Proof extension: perturbation-inspired Lyapunov



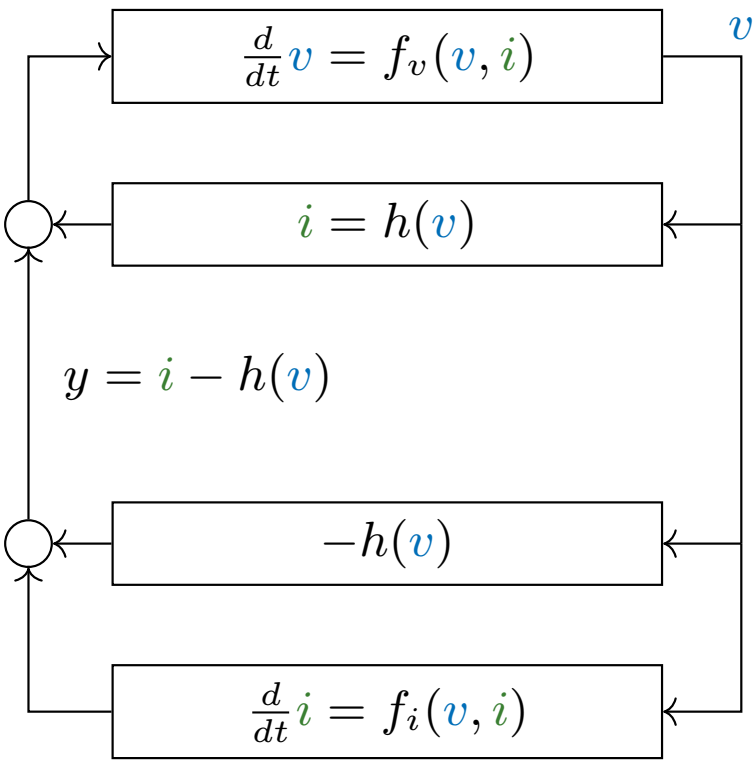
Individual Lyapunov functions

- slow system: $V(v)$ for $\frac{d}{dt}v = f_v(v, h(v))$
- fast system: $W(y)$ for $\frac{d}{dt}y = f_i(v, y + h(v))$
  where $\frac{d}{dt}v = 0$ & coordinate $y = i - h(v)$

Lyapunov function for the full system

- $\nu(x) = dW(i - h(v)) + (1 - d)V(v)$
  where $d \in [0, 1]$ is free convex coefficient
- $\frac{d}{dt}\nu(x)$ is decaying under stability condition

Proof extension: perturbation-inspired Lyapunov



Individual Lyapunov functions

- slow system: $V(v)$ for $\frac{d}{dt}v = f_v(v, h(v))$
- fast system: $W(y)$ for $\frac{d}{dt}y = f_i(v, y + h(v))$
  where $\frac{d}{dt}v = 0$ & coordinate $y = i - h(v)$

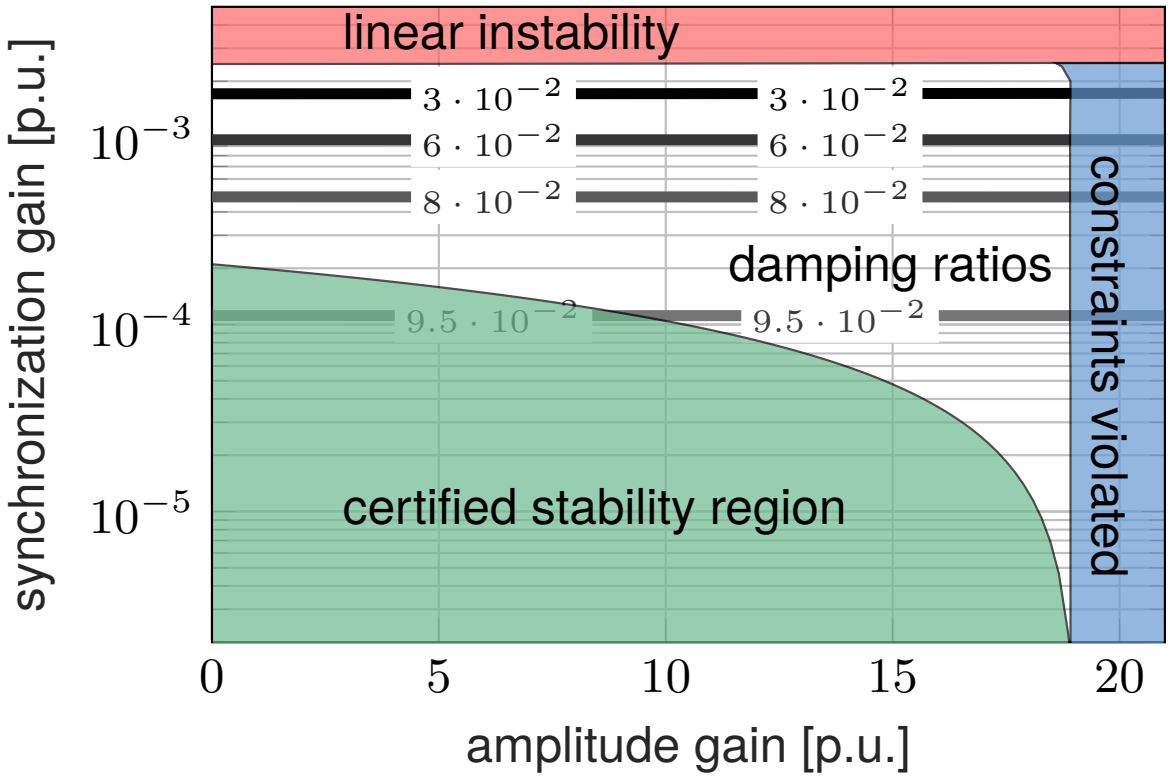
Lyapunov function for the full system

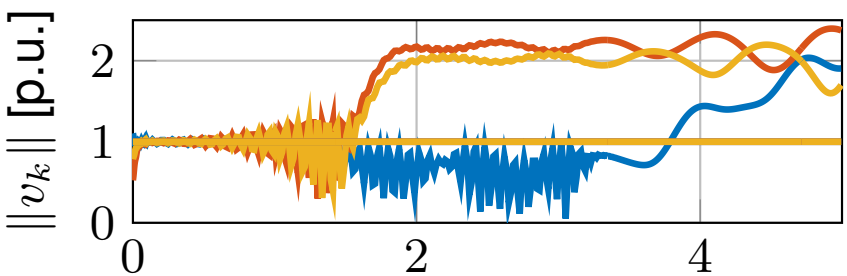
- $\nu(x) = dW(i - h(v)) + (1 - d)V(v)$
  where $d \in [0, 1]$ is free convex coefficient
- $\frac{d}{dt}\nu(x)$ is decaying under stability condition

Almost global asymptotic stability

- $\mathcal{T}' \cup \{\mathbb{0}\}$ globally attractive & $\mathcal{T}'$ stable
- $\mathcal{Z}_{\{\mathbb{0}\}}$ has measure zero

Evaluation of stability conditions





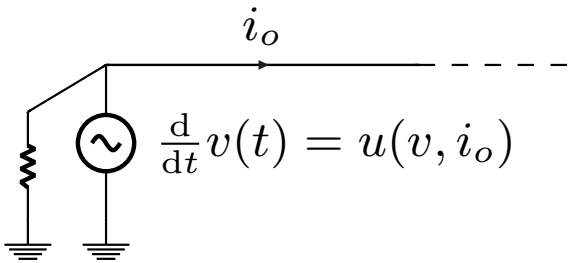
increase of control gains by factor 10

$\Rightarrow$ oscillations, overshoots, & instability

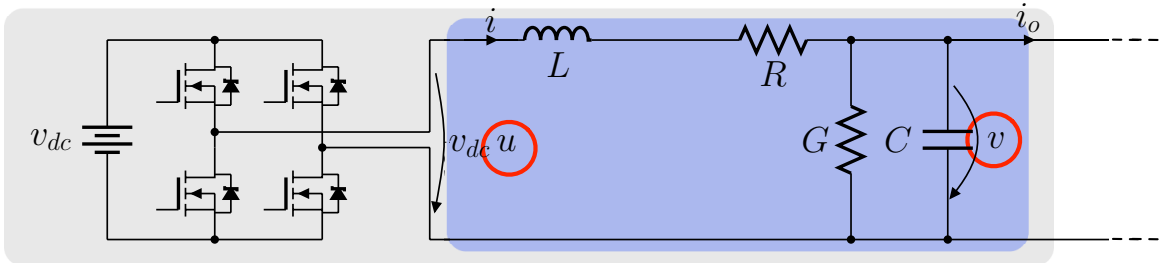
$\Rightarrow$ **conditions are highly accurate**

Dropping assumptions: detailed models

voltage source model:

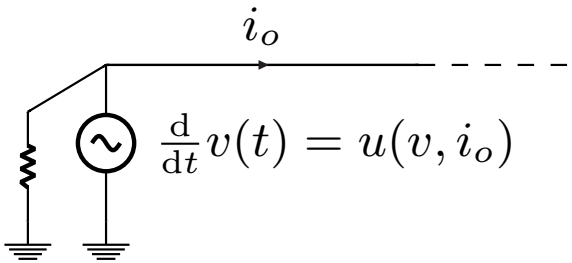


detailed converter model with LC filter:

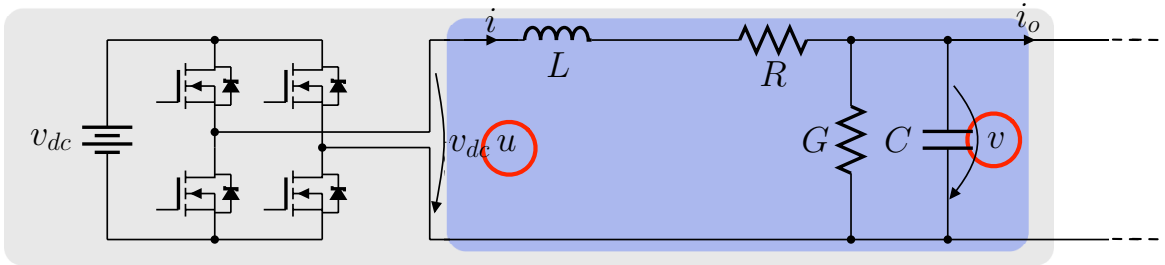


Dropping assumptions: detailed models

voltage source model:



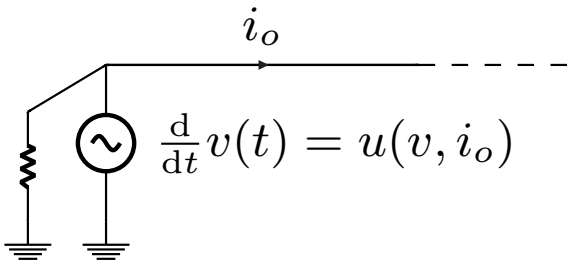
detailed converter model with LC filter:



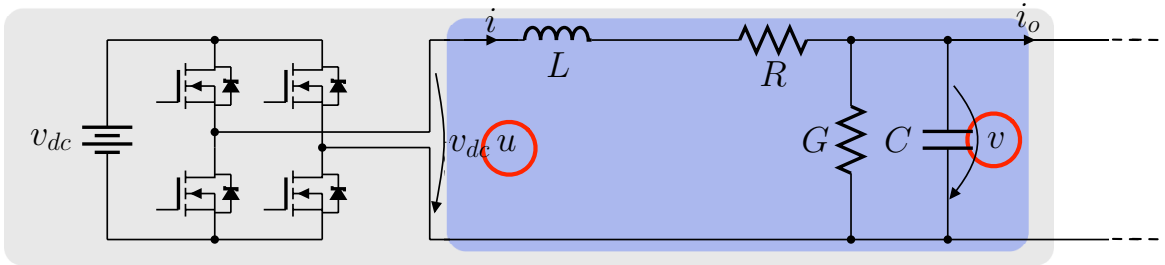
- **idea:** **invert** LC filter so that $v \approx v_{dc}u$

Dropping assumptions: detailed models

voltage source model:



detailed converter model with LC filter:

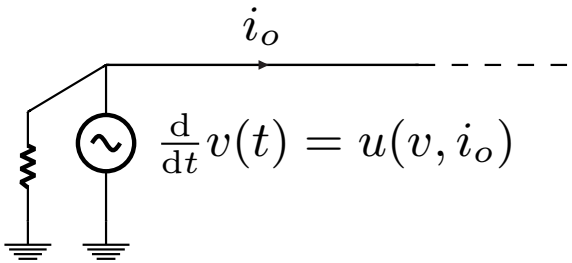


- **idea:** **invert** LC filter so that $v \approx v_{dc}u$

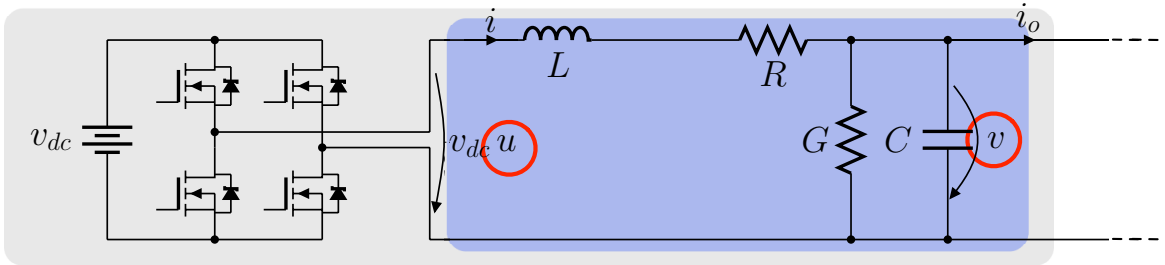
$\rightarrow$ **control:** perform robust inversion of LC filter via **cascaded PI**

Dropping assumptions: detailed models

voltage source model:



detailed converter model with LC filter:



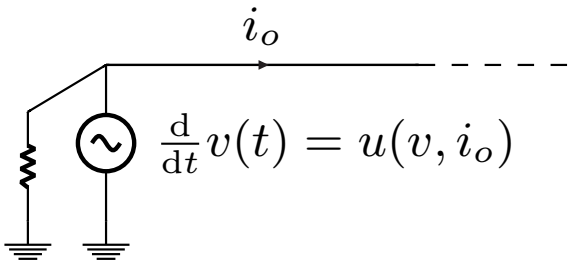
- **idea:** **invert** LC filter so that $v \approx v_{dc}u$

$\rightarrow$ **control:** perform robust inversion of LC filter via **cascaded PI**

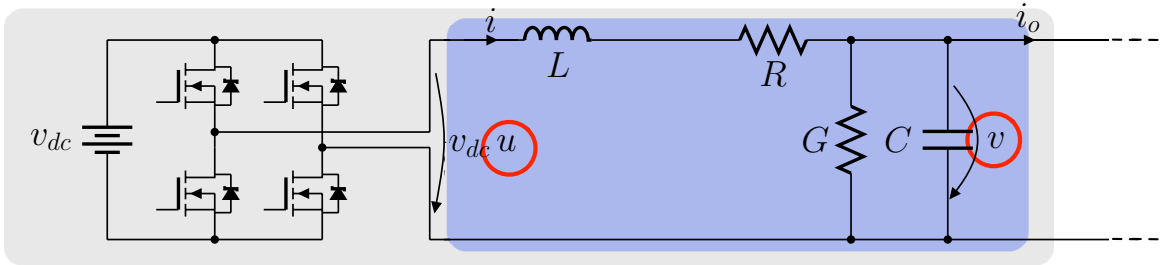
- **analysis:** repeat proof via **singular perturbation Lyapunov** functions

Dropping assumptions: detailed models

voltage source model:



detailed converter model with LC filter:



- **idea:** **invert** LC filter so that $v \approx v_{dc}u$

$\rightarrow$ **control:** perform robust inversion of LC filter via **cascaded PI**

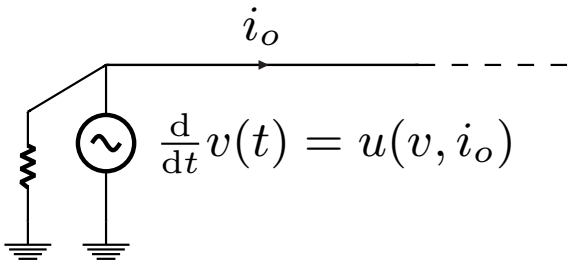
- **analysis:** repeat proof via **singular perturbation Lyapunov** functions

$\rightarrow$ **almost global stability** for sufficient time scale separation (quantifiable)

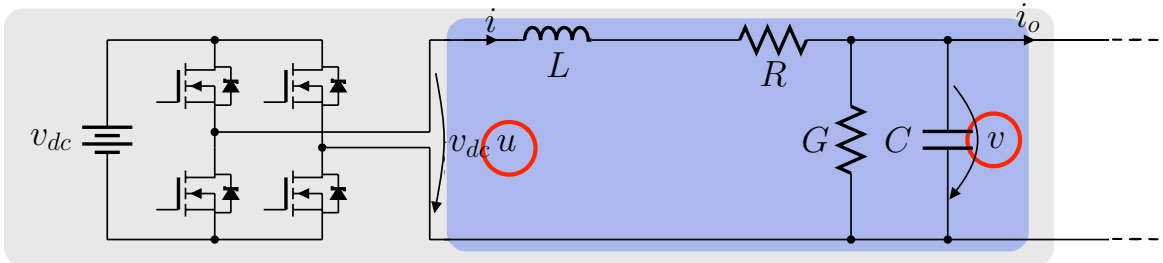
VOC model $<$ line dynamics $<$ voltage PI $<$ current PI

Dropping assumptions: detailed models

voltage source model:



detailed converter model with LC filter:



- **idea:** **invert** LC filter so that $v \approx v_{dc}u$

$\rightarrow$ **control:** perform robust inversion of LC filter via **cascaded PI**

- **analysis:** repeat proof via **singular perturbation Lyapunov** functions

$\rightarrow$ **almost global stability** for sufficient time scale separation (quantifiable)

$$\boxed{\text{VOC model} \;<\; \text{line dynamics} \;<\; \text{voltage PI} \;<\; \text{current PI}}$$

- . . . similar steps for control of v_{dc} in an even more detailed model

Details & initial conditions for further reading

A Lyapunov framework for nested dynamical systems on multiple time scales with application to converter-based power systems

Irina Subotić, Dominic Groß, Marcello Colombino, and Florian Dörfler

***Abstract*—In this work, we present a Lyapunov function framework for establishing stability with respect to a compact set of a nested interconnection of nonlinear dynamical systems ordered from slow to fast according to their convergence rates. The proposed approach explicitly considers more than two time scales, and does not require modeling multiple time scales via scalar time constants. Motivated by the technical results, we develop a novel control strategy for a grid-forming power converter that consists of an inner cascaded two-degree of freedom controller and dispatchable virtual oscillator control as a reference model. The resulting closed-loop converter-based AC power system is in the form of a nested system with multiple time scales. We apply our technical results to obtain explicit bounds on the controller**

The analysis in this paper is based on a recently developed Lyapunov characterization of almost global asymptotic stability with respect to a compact set presented in [8] that requires that the set of states that are unstable but attractive has zero Lebesgue measure. The technical contribution is twofold: first, we develop a Lyapunov characterization of unstable hyperbolic fixed points that have a region of attraction of measure zero; then we provide a Lyapunov framework that results in conditions under which the guarantees obtained by applying the aforementioned Lyapunov conditions in [8] to a reduced-order system translate to the full-order nested dynamical system. Our

High-Fidelity Model Order Reduction for Microgrids Stability Assessment

Petr Vorobev, *Member, IEEE*, Po-Hsu Huang, *Senior Member, IEEE*, Mohamed Al Hosani, *Member, IEEE*, James L. Kirtley, Jr., *Life Fellow, IEEE*, and Konstantin Turitsyn, *Member, IEEE*

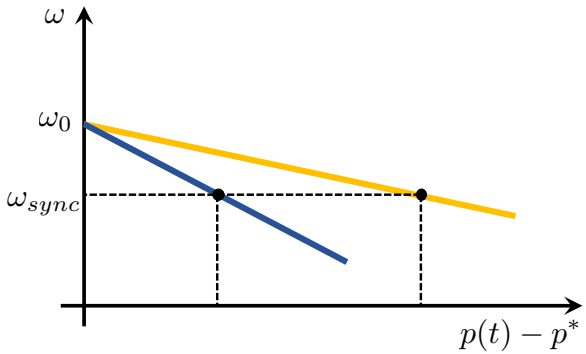
***Abstract*—Proper modeling of inverter-based microgrids is crucial for accurate assessment of stability boundaries. It has been recently realized that the stability conditions for such microgrids are significantly different from those known for large-scale power systems. In particular, the network dynamics, despite its fast nature, appears to have major influence on stability of slower modes. While detailed models are available, they are both computationally expensive and not transparent enough to provide an insight into the instability mechanisms and factors. In this paper, a computationally efficient and accurate reduced-order model is proposed for modeling inverter-based microgrids. The developed model has a structure similar to quasi-stationary model and at the same time properly accounts for the effects of network dynamics. The main factors affecting microgrid stability are analyzed using the devel-**

solution for geographically remote areas, where the interconnection to the main power grid is infeasible, but also are considered as an improvement for conventional distribution networks during their disconnection from the feeding substation [1]–[3]. While in grid-connected mode, the simplest and most commonly used method of operation is to set renewable sources to maximum power output with the grid's interconnection taking responsibility for any power imbalances. With the increasing share of distributed generation and, more importantly, in the islanded mode of operation, there is a need for proper control of individual inverters power output [1], [4]. The problem of designing proper controls for microgrids has been the subject of

Outline

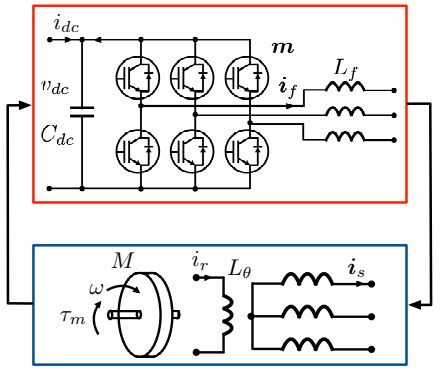
- Virtual Synchronous Machine Emulation
- Droop Control
- Matching Control
- Virtual Oscillator Control (VOC)
- Time-Scale Separation Issues
- Comparison

High-level comparison of grid-forming controllers



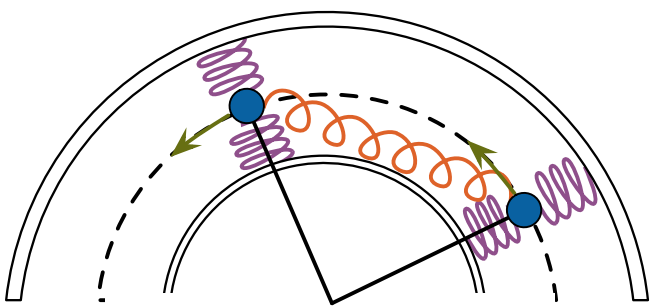
droop control

+ good performance near steady state
− relies on decoupling & small attraction basin



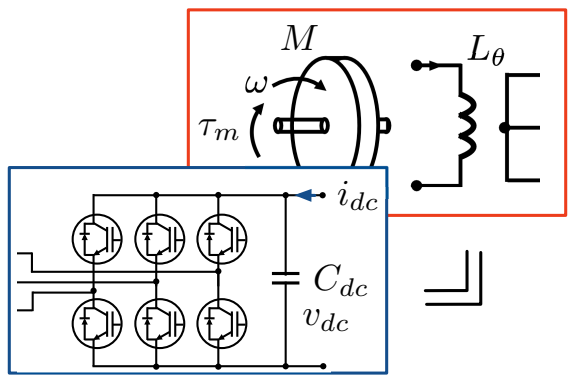
synchronous machine emulation

+ backward compatible in nominal case
− not resilient under large disturbances



virtual oscillator control

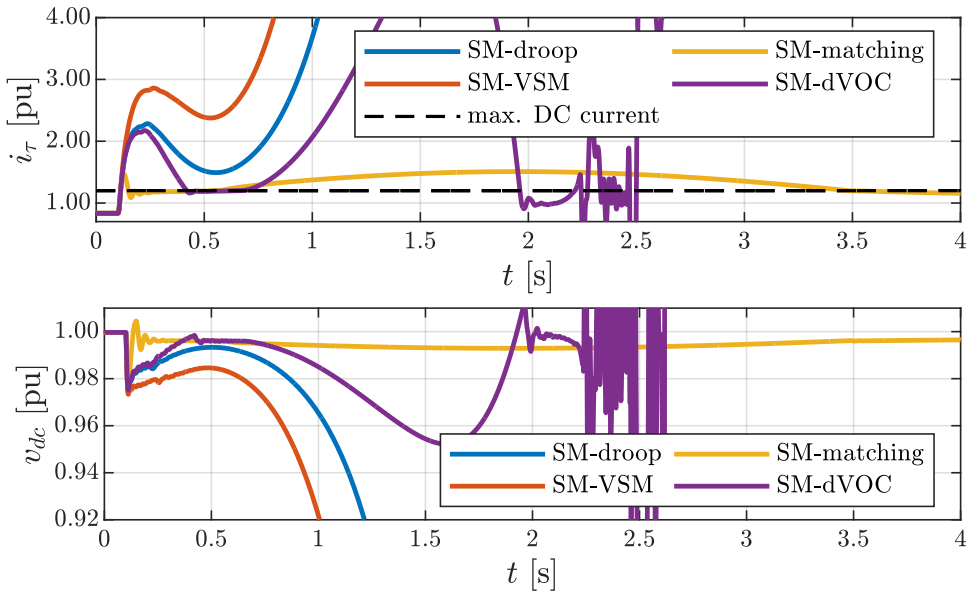
+ excellent large-signal behavior + local droop
− voc, droop, & vsm need strong dc source



matching control & duality

+ simple & robust
− slow ac performance

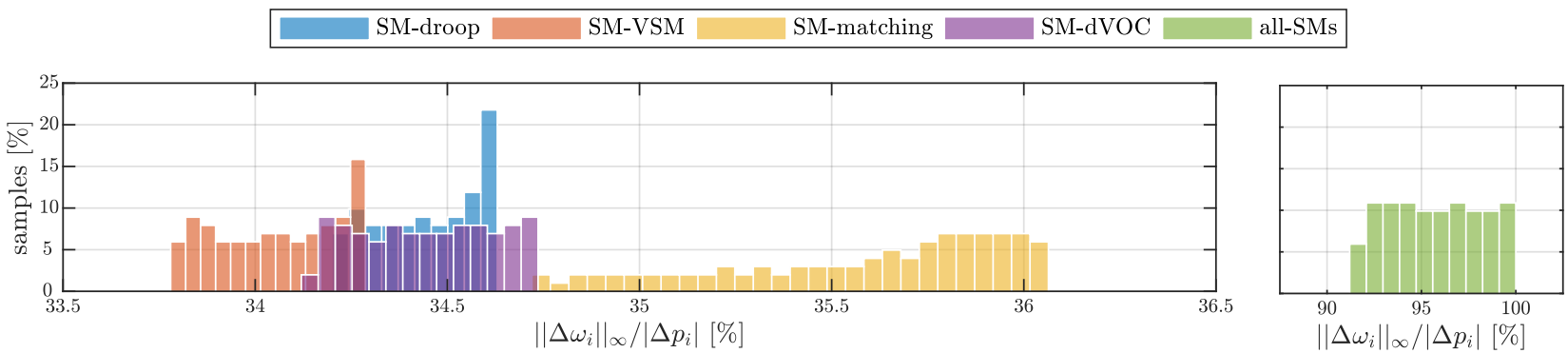
Some details of comparison study @AIT



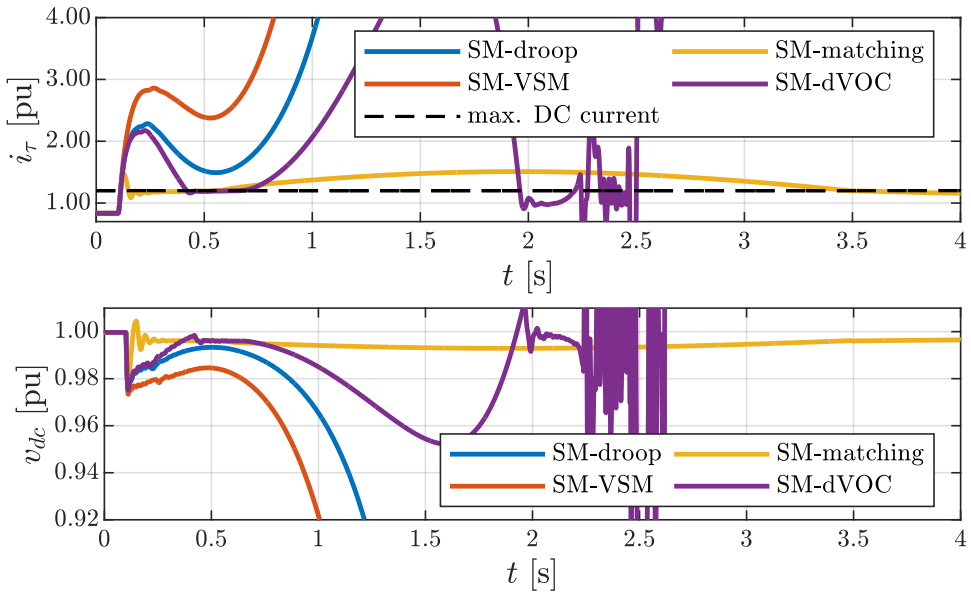
- **all perform well** nominally & under minor disturbances
- **relative resilience** :
  matching $>$ VOC $>$ droop $>$ virtual synchronous machine

Frequency Stability of Synchronous Machines and Grid-Forming Power Converters

Ali Tayyebi, Dominic Groß, *Member, IEEE*, Adolfo Anta, Friederich Kupzog and Florian Dörfler, *Member, IEEE*



Some details of comparison study @AIT

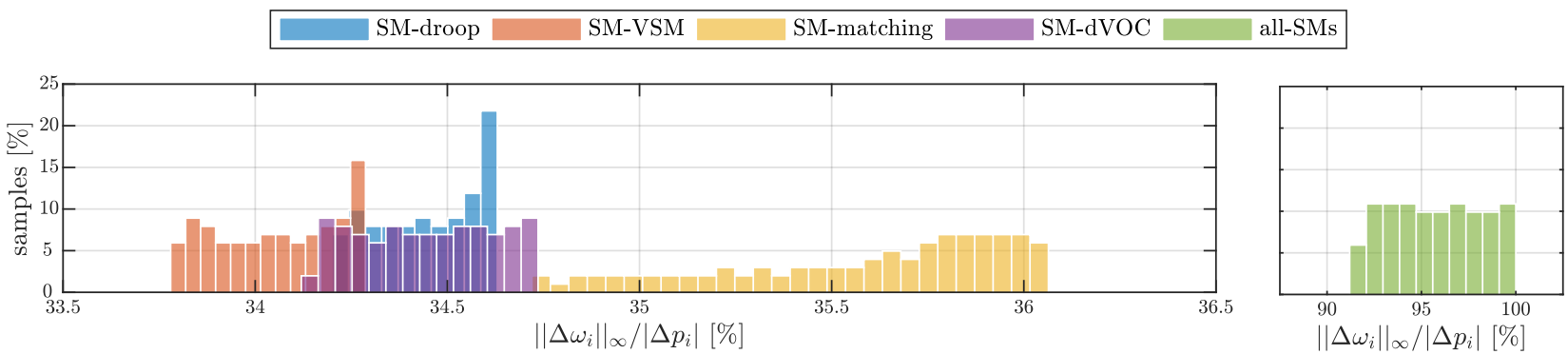


- **all perform well** nominally & under minor disturbances
- **relative resilience** :
  matching $>$ VOC $>$ droop $>$ virtual synchronous machine

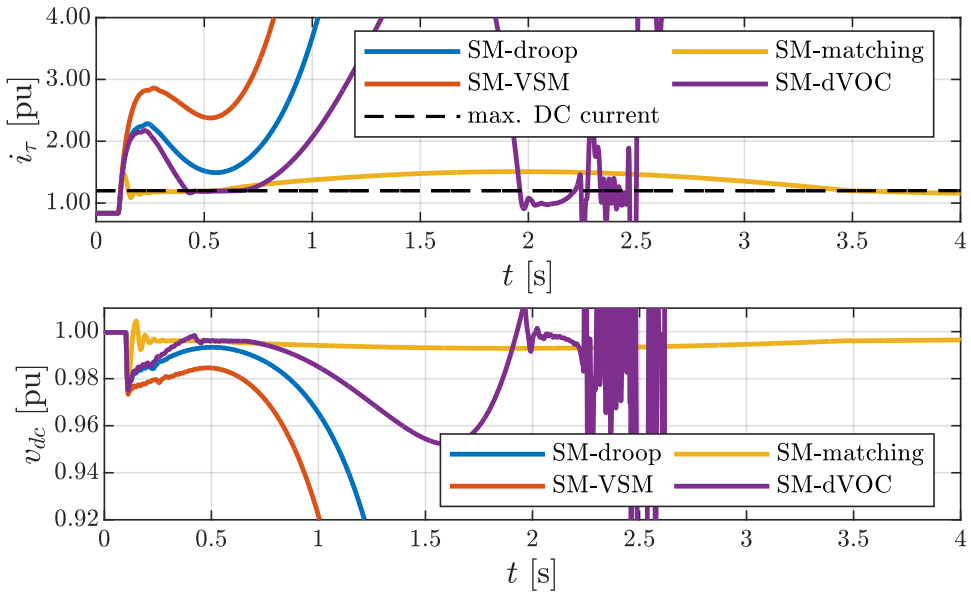
$\rightarrow$ it is a very poor strategy for a converter to emulate a flywheel

Frequency Stability of Synchronous Machines and Grid-Forming Power Converters

Ali Tayyebi, Dominic Groß, *Member, IEEE*, Adolfo Anta, Friederich Kupzog and Florian Dörfler, *Member, IEEE*



Some details of comparison study @AIT



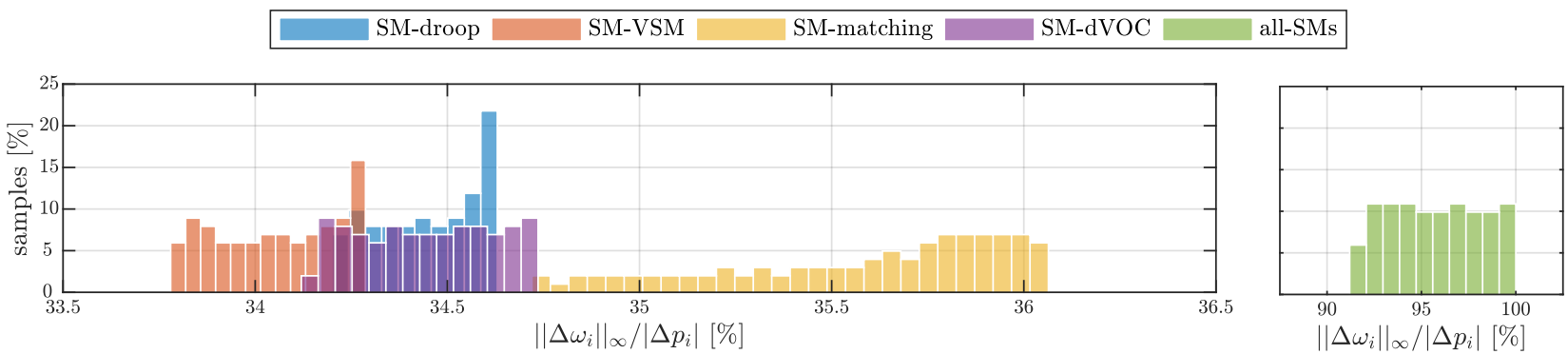
Frequency Stability of Synchronous Machines and Grid-Forming Power Converters

Ali Tayyebi, Dominic Groß, *Member, IEEE*, Adolfo Anta, Friederich Kupzog and Florian Dörfler, *Member, IEEE*

- **all perform well** nominally & under minor disturbances
- **relative resilience**:
  matching $>$ VOC $>$ droop $>$ virtual synchronous machine

$\rightarrow$ it is a very poor strategy for a converter to emulate a flywheel

- promising **hybrid control** directions: VOC + matching



Detailed comparison(s) (stopped collecting references at mid 2020)

Comparison of Virtual Oscillator and Droop Controlled Islanded Three-Phase Microgrids

Zhan Shi, *Member, IEEE*, Jiacheng Li, *Student Member, IEEE*, Hendra I. Nurdin, *Senior Member, IEEE*, and John E. Fletcher, *Senior Member, IEEE*

Comparison of Virtual Oscillator and Droop Control

Brian Johnson, Miguel Rodriguez
Power Systems Engineering Center
National Renewable Energy Laboratory
Golden, CO 80401
Email: brian.johnson@nrel.gov, miguelrg@gmail.com

Mohit Sinha, Sairaj Dhople
Department of Electrical & Computer Engineering
University of Minnesota
Minneapolis, MN 55455
Email: {sinha052,sdhople}@umn.edu

Similarities between Virtual Oscillator Controlled and Droop Controlled Three-Phase Inverters

Zhan Shi, Hendra I. Nurdin, John E. Fletcher, Jiacheng Li
School of Electrical Engineering and Telecommunications, UNSW Sydney, NSW, 2052, Australia
Email: zhan.shi@unsw.edu.au, h.nurdin@unsw.edu.au, john.fletcher@unsw.edu.au, jiacheng.li2@unsw.edu.au

Comparative Transient Stability Assessment of Droop and Dispatchable Virtual Oscillator Controlled Grid-Connected Inverters

Hui Yu, *Student Member, IEEE,* M A Awal, *Student Member, IEEE,* Hao Tu, *Student Member, IEEE,* Iqbal Husain, *Fellow, IEEE* and Srdjan Lukic, *Senior Member, IEEE,*

Frequency Stability of Synchronous Machines and Grid-Forming Power Converters

Ali Tayyebi, Dominic Groß, *Member, IEEE*, Adolfo Anta, Friederich Kupzog and Florian Dörfler, *Member, IEEE*

GRID-FORMING CONVERTERS – INEVITABILITY, CONTROL STRATEGIES AND CHALLENGES IN FUTURE GRIDS APPLICATION

Ali TAYYEBI
AIT and ETH Zürich – Austria

Florian DÖRFLER
ETH Zürich – Switzerland

Friederich KUPZOG
Austrian Institute of Technology – Austria

Comparison of Droop Control and Virtual Oscillator Control Realized by Andronov-Hopf Dynamics

Minghui Lu*, Victor Purba†, Sairaj Dhople†, Brian Johnson*

Transient response comparison of virtual oscillator controlled and droop controlled three-phase inverters under load changes

Zhan Shi[1] ✉, Jiacheng Li[1], Hendra I. Nurdin[1], John E. Fletcher[1]
[1]School of Electrical Engineering and Telecommunications, UNSW Sydney, UNSW, NSW, 2052, Australia
✉E-mail: zhan.shi@unsw.edu.au

Simulation-based study of novel control strategies for inverters in low-inertia system: grid-forming and grid-following

Author: Alessandro Crivellaro

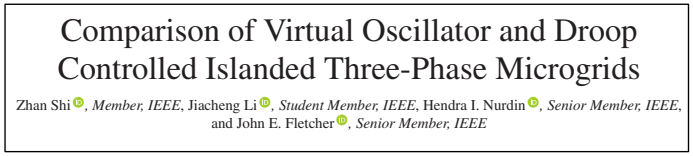
Mathias Melby

Comparison of virtual oscillator control and droop control in an inverter-based stand-alone microgrid

Grid-Forming Converters control based on DC voltage feedback

Yuan Gao[a], Hai-Peng Ren[a], Jie Li[a]

Detailed comparison(s) (stopped collecting references at mid 2020)



Comparison of Virtual Oscillator and Droop Controlled Islanded Three-Phase Microgrids

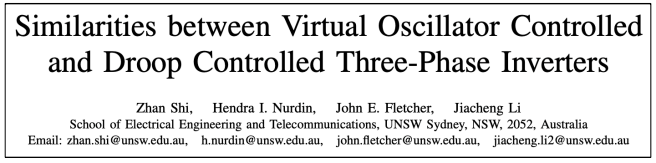
Zhan Shi, *Member, IEEE*, Jiacheng Li, *Student Member, IEEE*, Hendra I. Nurdin, *Senior Member, IEEE*, and John E. Fletcher, *Senior Member, IEEE*



Comparison of Virtual Oscillator and Droop Control

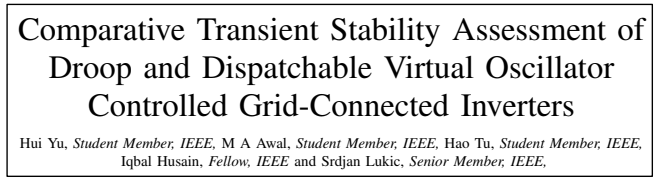
Brian Johnson, Miguel Rodriguez
Power Systems Engineering Center
National Renewable Energy Laboratory
Golden, CO 80401
Email: brian.johnson@nrel.gov, miguelrg@gmail.com

Mohit Sinha, Sairaj Dhople
Department of Electrical & Computer Engineering
University of Minnesota
Minneapolis, MN 55455
Email: {sinha052,sdhople}@umn.edu



Similarities between Virtual Oscillator Controlled and Droop Controlled Three-Phase Inverters

Zhan Shi, Hendra I. Nurdin, John E. Fletcher, Jiacheng Li
School of Electrical Engineering and Telecommunications, UNSW Sydney, NSW, 2052, Australia
Email: zhan.shi@unsw.edu.au, h.nurdin@unsw.edu.au, john.fletcher@unsw.edu.au, jiacheng.li2@unsw.edu.au



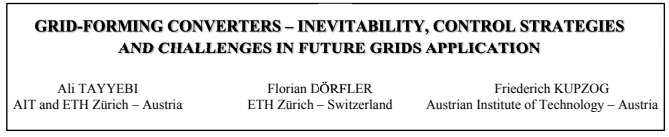
Comparative Transient Stability Assessment of Droop and Dispatchable Virtual Oscillator Controlled Grid-Connected Inverters

Hui Yu, *Student Member, IEEE,* M A Awal, *Student Member, IEEE,* Hao Tu, *Student Member, IEEE,* Iqbal Husain, *Fellow, IEEE* and Srdjan Lukic, *Senior Member, IEEE,*



Frequency Stability of Synchronous Machines and Grid-Forming Power Converters

Ali Tayyebi, Dominic Groß, *Member, IEEE*, Adolfo Anta, Friederich Kupzog and Florian Dörfler, *Member, IEEE*



GRID-FORMING CONVERTERS – INEVITABILITY, CONTROL STRATEGIES AND CHALLENGES IN FUTURE GRIDS APPLICATION

Ali TAYYEBI
AIT and ETH Zürich – Austria

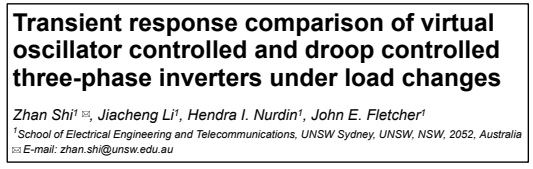
Florian DÖRFLER
ETH Zürich – Switzerland

Friederich KUPZOG
Austrian Institute of Technology – Austria



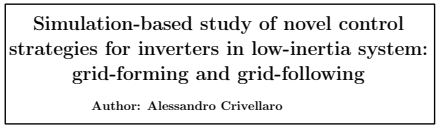
Comparison of Droop Control and Virtual Oscillator Control Realized by Andronov-Hopf Dynamics

Minghui Lu*, Victor Purba†, Sairaj Dhople†, Brian Johnson*



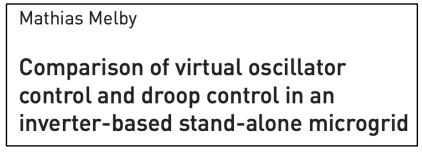
Transient response comparison of virtual oscillator controlled and droop controlled three-phase inverters under load changes

Zhan Shi[1], Jiacheng Li[1], Hendra I. Nurdin[1], John E. Fletcher[1]
[1]School of Electrical Engineering and Telecommunications, UNSW Sydney, UNSW, NSW, 2052, Australia
E-mail: zhan.shi@unsw.edu.au



Simulation-based study of novel control strategies for inverters in low-inertia system: grid-forming and grid-following

Author: Alessandro Crivellaro



Mathias Melby

Comparison of virtual oscillator control and droop control in an inverter-based stand-alone microgrid

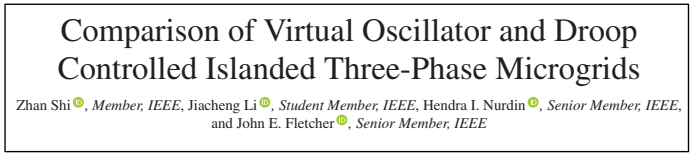


Grid-Forming Converters control based on DC voltage feedback

Yuan Gao[a], Hai-Peng Ren[a], Jie Li[a]

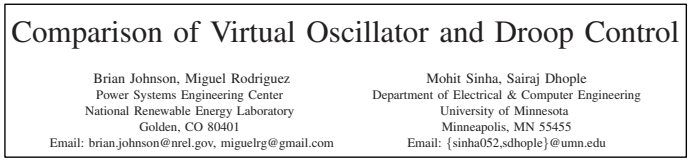
- **identical steady-state & similar small-signal behavior** (after tuning)

Detailed comparison(s) (stopped collecting references at mid 2020)



Comparison of Virtual Oscillator and Droop Controlled Islanded Three-Phase Microgrids

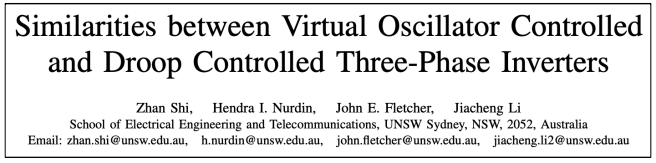
Zhan Shi, *Member, IEEE*, Jiacheng Li, *Student Member, IEEE*, Hendra I. Nurdin, *Senior Member, IEEE*, and John E. Fletcher, *Senior Member, IEEE*



Comparison of Virtual Oscillator and Droop Control

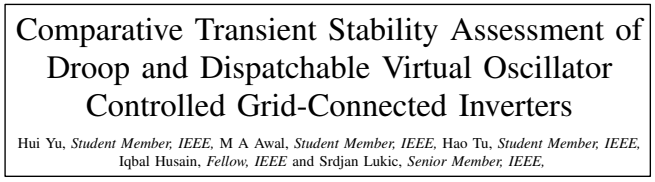
Brian Johnson, Miguel Rodriguez
Power Systems Engineering Center
National Renewable Energy Laboratory
Golden, CO 80401
Email: brian.johnson@nrel.gov, miguelrg@gmail.com

Mohit Sinha, Sairaj Dhople
Department of Electrical & Computer Engineering
University of Minnesota
Minneapolis, MN 55455
Email: {sinha052,sdhople}@umn.edu



Similarities between Virtual Oscillator Controlled and Droop Controlled Three-Phase Inverters

Zhan Shi, Hendra I. Nurdin, John E. Fletcher, Jiacheng Li
School of Electrical Engineering and Telecommunications, UNSW Sydney, NSW, 2052, Australia
Email: zhan.shi@unsw.edu.au, h.nurdin@unsw.edu.au, john.fletcher@unsw.edu.au, jiacheng.li2@unsw.edu.au



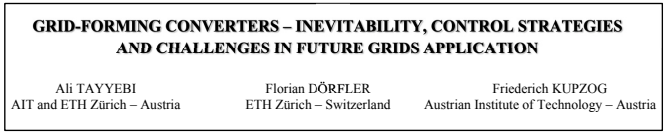
Comparative Transient Stability Assessment of Droop and Dispatchable Virtual Oscillator Controlled Grid-Connected Inverters

Hui Yu, *Student Member, IEEE,* M A Awal, *Student Member, IEEE,* Hao Tu, *Student Member, IEEE,* Iqbal Husain, *Fellow, IEEE* and Srdjan Lukic, *Senior Member, IEEE,*



Frequency Stability of Synchronous Machines and Grid-Forming Power Converters

Ali Tayyebi, Dominic Groß, *Member, IEEE*, Adolfo Anta, Friederich Kupzog and Florian Dörfler, *Member, IEEE*



GRID-FORMING CONVERTERS – INEVITABILITY, CONTROL STRATEGIES AND CHALLENGES IN FUTURE GRIDS APPLICATION

Ali TAYYEBI
AIT and ETH Zürich – Austria

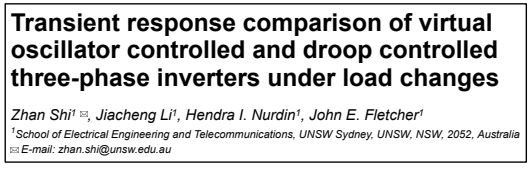
Florian DÖRFLER
ETH Zürich – Switzerland

Friederich KUPZOG
Austrian Institute of Technology – Austria



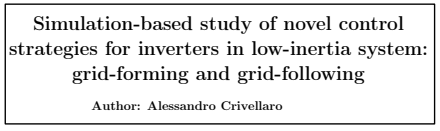
Comparison of Droop Control and Virtual Oscillator Control Realized by Andronov-Hopf Dynamics

Minghui Lu*, Victor Purba†, Sairaj Dhople†, Brian Johnson*



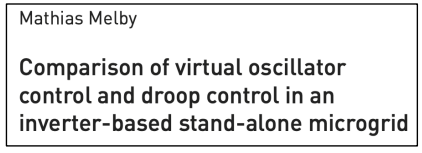
Transient response comparison of virtual oscillator controlled and droop controlled three-phase inverters under load changes

Zhan Shi[1], Jiacheng Li[1], Hendra I. Nurdin[1], John E. Fletcher[1]
[1]School of Electrical Engineering and Telecommunications, UNSW Sydney, UNSW, NSW, 2052, Australia
E-mail: zhan.shi@unsw.edu.au



Simulation-based study of novel control strategies for inverters in low-inertia system: grid-forming and grid-following

Author: Alessandro Crivellaro



Mathias Melby

Comparison of virtual oscillator control and droop control in an inverter-based stand-alone microgrid

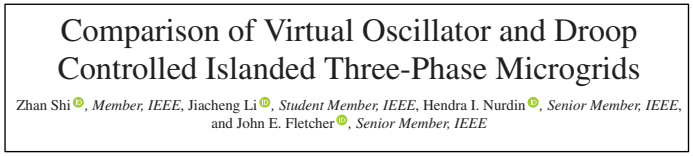


Grid-Forming Converters control based on DC voltage feedback

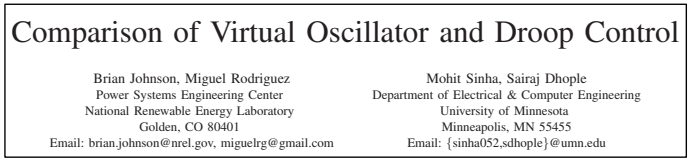
Yuan Gao[a], Hai-Peng Ren[a], Jie Li[a]

- **identical steady-state & similar small-signal behavior** (after tuning)
- **virtual synchronous machine** has poor transients (converter $\neq$ flywheel)

Detailed comparison(s) (stopped collecting references at mid 2020)

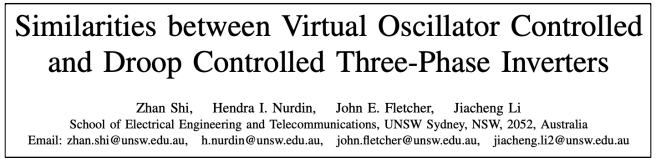

Comparison of Virtual Oscillator and Droop Controlled Islanded Three-Phase Microgrids

Zhan Shi, *Member, IEEE*, Jiacheng Li, *Student Member, IEEE*, Hendra I. Nurdin, *Senior Member, IEEE*, and John E. Fletcher, *Senior Member, IEEE*

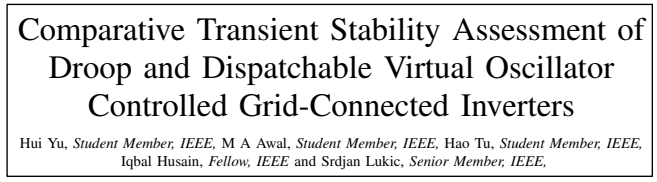

Comparison of Virtual Oscillator and Droop Control

Brian Johnson, Miguel Rodriguez
Power Systems Engineering Center
National Renewable Energy Laboratory
Golden, CO 80401
Email: brian.johnson@nrel.gov, miguelrg@gmail.com

Mohit Sinha, Sairaj Dhople
Department of Electrical & Computer Engineering
University of Minnesota
Minneapolis, MN 55455
Email: {sinha052,sdhople}@umn.edu


Similarities between Virtual Oscillator Controlled and Droop Controlled Three-Phase Inverters

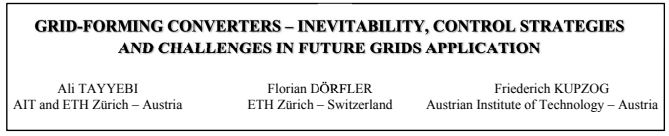
Zhan Shi, Hendra I. Nurdin, John E. Fletcher, Jiacheng Li
School of Electrical Engineering and Telecommunications, UNSW Sydney, NSW, 2052, Australia
Email: zhan.shi@unsw.edu.au, h.nurdin@unsw.edu.au, john.fletcher@unsw.edu.au, jiacheng.li2@unsw.edu.au


Comparative Transient Stability Assessment of Droop and Dispatchable Virtual Oscillator Controlled Grid-Connected Inverters

Hui Yu, *Student Member, IEEE,* M A Awal, *Student Member, IEEE,* Hao Tu, *Student Member, IEEE,* Iqbal Husain, *Fellow, IEEE* and Srdjan Lukic, *Senior Member, IEEE,*


Frequency Stability of Synchronous Machines and Grid-Forming Power Converters

Ali Tayyebi, Dominic Groß, *Member, IEEE*, Adolfo Anta, Friederich Kupzog and Florian Dörfler, *Member, IEEE*


GRID-FORMING CONVERTERS – INEVITABILITY, CONTROL STRATEGIES AND CHALLENGES IN FUTURE GRIDS APPLICATION

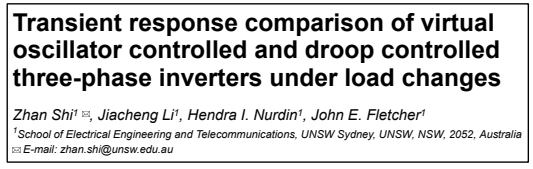
Ali TAYYEBI
AIT and ETH Zürich – Austria

Florian DÖRFLER
ETH Zürich – Switzerland

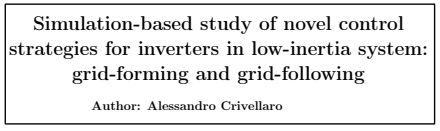
Friederich KUPZOG
Austrian Institute of Technology – Austria


Comparison of Droop Control and Virtual Oscillator Control Realized by Andronov-Hopf Dynamics

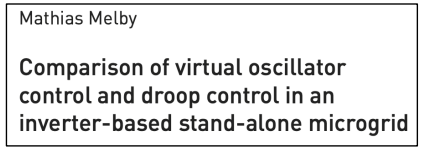
Minghui Lu*, Victor Purba†, Sairaj Dhople†, Brian Johnson*


Transient response comparison of virtual oscillator controlled and droop controlled three-phase inverters under load changes

Zhan Shi[1] ✉, Jiacheng Li[1], Hendra I. Nurdin[1], John E. Fletcher[1]
[1]School of Electrical Engineering and Telecommunications, UNSW Sydney, UNSW, NSW, 2052, Australia
✉E-mail: zhan.shi@unsw.edu.au


Simulation-based study of novel control strategies for inverters in low-inertia system: grid-forming and grid-following

Author: Alessandro Crivellaro


Mathias Melby

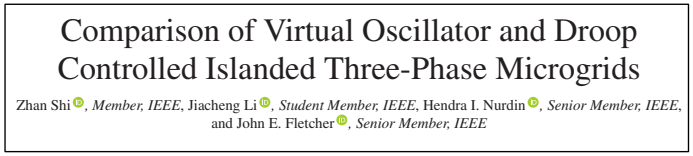
Comparison of virtual oscillator control and droop control in an inverter-based stand-alone microgrid


Grid-Forming Converters control based on DC voltage feedback

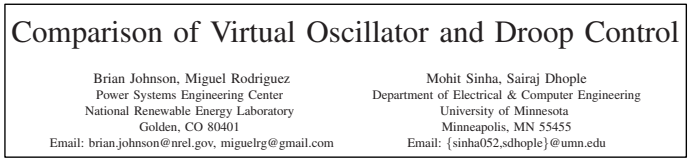
Yuan Gao[a], Hai-Peng Ren[a], Jie Li[a]

- **identical steady-state & similar small-signal behavior** (after tuning)
- **virtual synchronous machine** has poor transients (converter $\neq$ flywheel)
- **VOC has best large-signal behavior**: stability, post-fault-response, ...

Detailed comparison(s) (stopped collecting references at mid 2020)

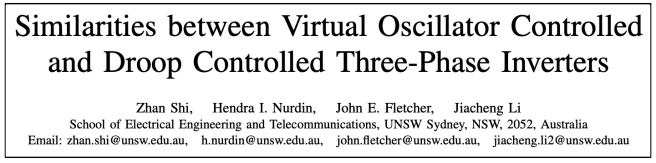

Comparison of Virtual Oscillator and Droop Controlled Islanded Three-Phase Microgrids

Zhan Shi, *Member, IEEE*, Jiacheng Li, *Student Member, IEEE*, Hendra I. Nurdin, *Senior Member, IEEE*, and John E. Fletcher, *Senior Member, IEEE*

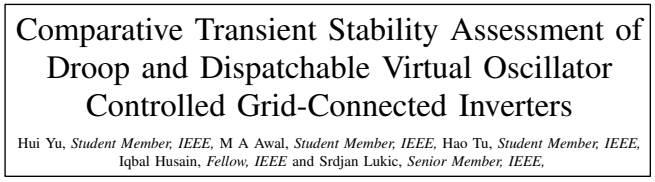

Comparison of Virtual Oscillator and Droop Control

Brian Johnson, Miguel Rodriguez
Power Systems Engineering Center
National Renewable Energy Laboratory
Golden, CO 80401
Email: brian.johnson@nrel.gov, miguelrg@gmail.com

Mohit Sinha, Sairaj Dhople
Department of Electrical & Computer Engineering
University of Minnesota
Minneapolis, MN 55455
Email: {sinha052,sdhople}@umn.edu


Similarities between Virtual Oscillator Controlled and Droop Controlled Three-Phase Inverters

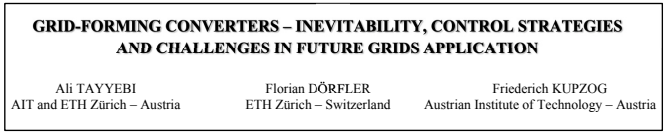
Zhan Shi, Hendra I. Nurdin, John E. Fletcher, Jiacheng Li
School of Electrical Engineering and Telecommunications, UNSW Sydney, NSW, 2052, Australia
Email: zhan.shi@unsw.edu.au, h.nurdin@unsw.edu.au, john.fletcher@unsw.edu.au, jiacheng.li2@unsw.edu.au


Comparative Transient Stability Assessment of Droop and Dispatchable Virtual Oscillator Controlled Grid-Connected Inverters

Hui Yu, *Student Member, IEEE,* M A Awal, *Student Member, IEEE,* Hao Tu, *Student Member, IEEE,* Iqbal Husain, *Fellow, IEEE* and Srdjan Lukic, *Senior Member, IEEE,*


Frequency Stability of Synchronous Machines and Grid-Forming Power Converters

Ali Tayyebi, Dominic Groß, *Member, IEEE*, Adolfo Anta, Friederich Kupzog and Florian Dörfler, *Member, IEEE*


GRID-FORMING CONVERTERS – INEVITABILITY, CONTROL STRATEGIES AND CHALLENGES IN FUTURE GRIDS APPLICATION

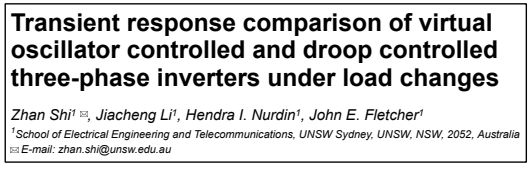
Ali TAYYEBI
AIT and ETH Zürich – Austria

Florian DÖRFLER
ETH Zürich – Switzerland

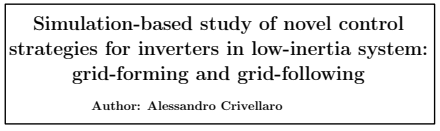
Friederich KUPZOG
Austrian Institute of Technology – Austria


Comparison of Droop Control and Virtual Oscillator Control Realized by Andronov-Hopf Dynamics

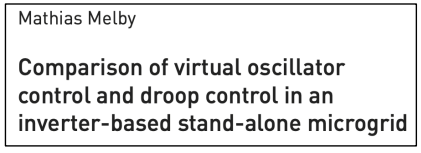
Minghui Lu*, Victor Purba†, Sairaj Dhople†, Brian Johnson*


Transient response comparison of virtual oscillator controlled and droop controlled three-phase inverters under load changes

Zhan Shi[1] ✉, Jiacheng Li[1], Hendra I. Nurdin[1], John E. Fletcher[1]
[1]School of Electrical Engineering and Telecommunications, UNSW Sydney, UNSW, NSW, 2052, Australia
✉E-mail: zhan.shi@unsw.edu.au


Simulation-based study of novel control strategies for inverters in low-inertia system: grid-forming and grid-following

Author: Alessandro Crivellaro


Mathias Melby

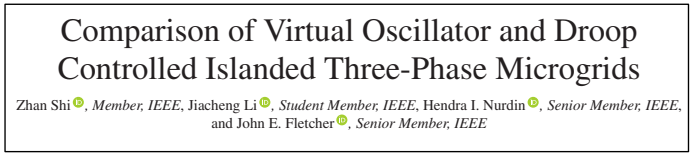
Comparison of virtual oscillator control and droop control in an inverter-based stand-alone microgrid


Grid-Forming Converters control based on DC voltage feedback

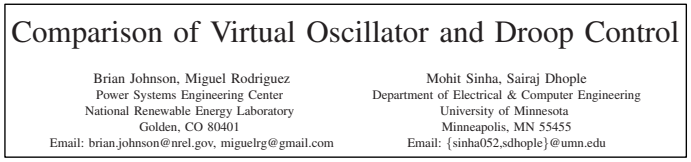
Yuan Gao[a], Hai-Peng Ren[a], Jie Li[a]

- **identical steady-state & similar small-signal behavior** (after tuning)
- **virtual synchronous machine** has poor transients (converter $\neq$ flywheel)
- **VOC has best large-signal behavior**: stability, post-fault-response, ...
- **matching control** $\omega \sim v_{dc}$ **is most robust** though with slow AC dynamics

Detailed comparison(s) (stopped collecting references at mid 2020)

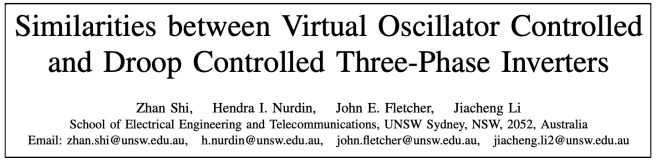

Comparison of Virtual Oscillator and Droop Controlled Islanded Three-Phase Microgrids

Zhan Shi, *Member, IEEE*, Jiacheng Li, *Student Member, IEEE*, Hendra I. Nurdin, *Senior Member, IEEE*, and John E. Fletcher, *Senior Member, IEEE*

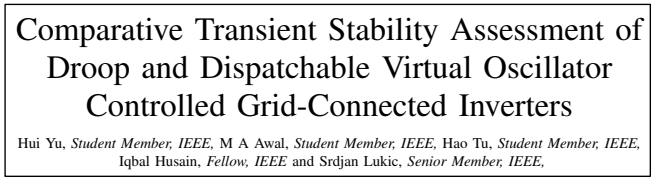

Comparison of Virtual Oscillator and Droop Control

Brian Johnson, Miguel Rodriguez
Power Systems Engineering Center
National Renewable Energy Laboratory
Golden, CO 80401
Email: brian.johnson@nrel.gov, miguelrg@gmail.com

Mohit Sinha, Sairaj Dhople
Department of Electrical & Computer Engineering
University of Minnesota
Minneapolis, MN 55455
Email: {sinha052,sdhople}@umn.edu


Similarities between Virtual Oscillator Controlled and Droop Controlled Three-Phase Inverters

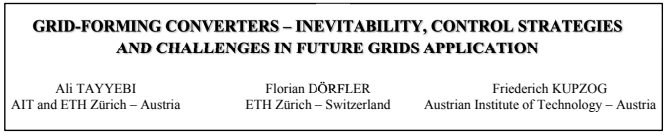
Zhan Shi, Hendra I. Nurdin, John E. Fletcher, Jiacheng Li
School of Electrical Engineering and Telecommunications, UNSW Sydney, NSW, 2052, Australia
Email: zhan.shi@unsw.edu.au, h.nurdin@unsw.edu.au, john.fletcher@unsw.edu.au, jiacheng.li2@unsw.edu.au


Comparative Transient Stability Assessment of Droop and Dispatchable Virtual Oscillator Controlled Grid-Connected Inverters

Hui Yu, *Student Member, IEEE,* M A Awal, *Student Member, IEEE,* Hao Tu, *Student Member, IEEE,* Iqbal Husain, *Fellow, IEEE* and Srdjan Lukic, *Senior Member, IEEE,*


Frequency Stability of Synchronous Machines and Grid-Forming Power Converters

Ali Tayyebi, Dominic Groß, *Member, IEEE*, Adolfo Anta, Friederich Kupzog and Florian Dörfler, *Member, IEEE*


GRID-FORMING CONVERTERS – INEVITABILITY, CONTROL STRATEGIES AND CHALLENGES IN FUTURE GRIDS APPLICATION

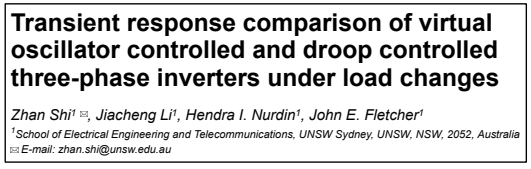
Ali TAYYEBI
AIT and ETH Zürich – Austria

Florian DÖRFLER
ETH Zürich – Switzerland

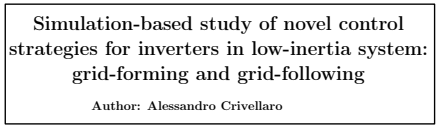
Friederich KUPZOG
Austrian Institute of Technology – Austria


Comparison of Droop Control and Virtual Oscillator Control Realized by Andronov-Hopf Dynamics

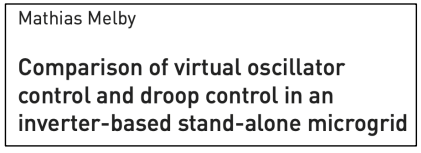
Minghui Lu*, Victor Purba†, Sairaj Dhople†, Brian Johnson*


Transient response comparison of virtual oscillator controlled and droop controlled three-phase inverters under load changes

Zhan Shi[1] ✉, Jiacheng Li[1], Hendra I. Nurdin[1], John E. Fletcher[1]
[1]School of Electrical Engineering and Telecommunications, UNSW Sydney, UNSW, NSW, 2052, Australia
✉E-mail: zhan.shi@unsw.edu.au


Simulation-based study of novel control strategies for inverters in low-inertia system: grid-forming and grid-following

Author: Alessandro Crivellaro


Mathias Melby

Comparison of virtual oscillator control and droop control in an inverter-based stand-alone microgrid


Grid-Forming Converters control based on DC voltage feedback

Yuan Gao[a], Hai-Peng Ren[a], Jie Li[a]

- **identical steady-state & similar small-signal behavior** (after tuning)
- **virtual synchronous machine** has poor transients (converter $\neq$ flywheel)
- **VOC has best large-signal behavior**: stability, post-fault-response, ...
- **matching control** $\omega \sim v_{dc}$ **is most robust** though with slow AC dynamics
- ...comparison suggests **hybrid VOC + matching control** direction

Optimal multivariable linear grid-forming control

$$\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} = \mathbb{K}(s) \begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix}$$

- inputs: modulation, dc-power supply, & inner references
- outputs: (nonlinear) state tracking errors

[Huang, Xin, & Dörfler, '20],

[Chen, Zhou, Tayybei, Prieto, Dörfler, & Blaabjerg, '21]

Optimal multivariable linear grid-forming control

$$\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} = \mathbb{K}(s) \begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix}$$

- inputs: modulation, dc-power supply, & inner references
- outputs: (nonlinear) state tracking errors

→ **MIMO transfer matrix** relating error signals **includes all other controls** (e.g., droop, VOC, VSM, matching, . . .)

- **optimal/robust linear design** via $\mathcal{H}_2$ / $\mathcal{H}_\infty$ & nonlinear implementation
- **forming/following mode** enforced by small-signal Bode characterization
- **linear stability** under interconnection

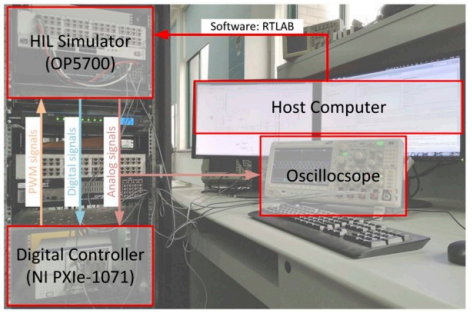
[Huang, Xin, & Dörfler, '20],

[Chen, Zhou, Tayybei, Prieto, Dörfler, & Blaabjerg, '21]

Optimal multivariable linear grid-forming control

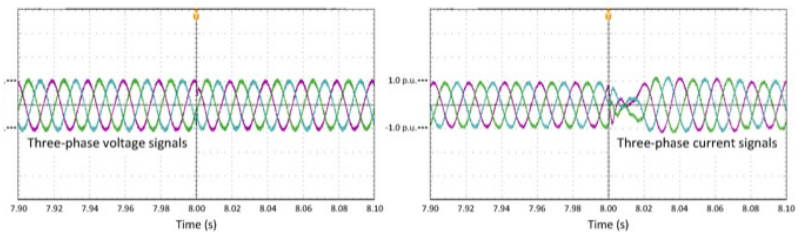
$$\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} = \mathbb{K}(s) \begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix}$$

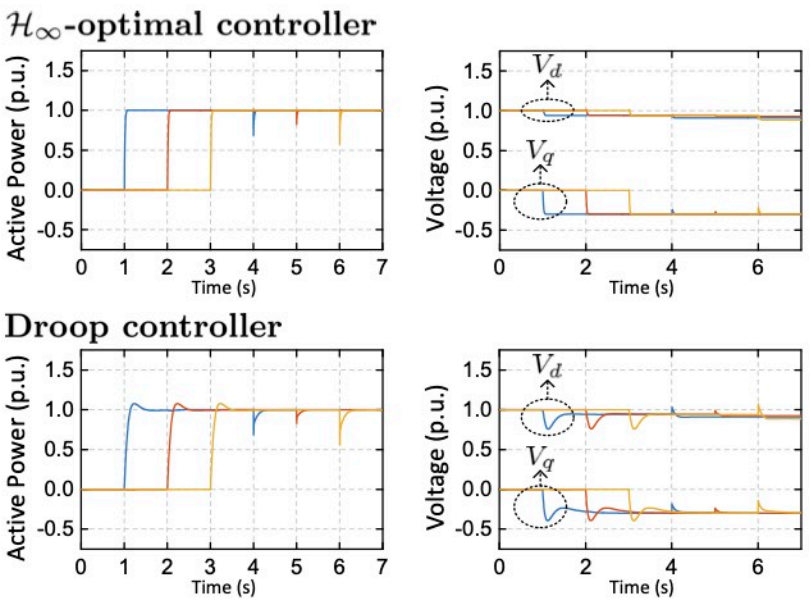
- inputs: modulation, dc-power supply, & inner references
- outputs: (nonlinear) state tracking errors



→ **MIMO transfer matrix** relating error signals **includes all other controls** (e.g., droop, VOC, VSM, matching, . . .)

- **optimal/robust linear design** via $\mathcal{H}_2$ / $\mathcal{H}_\infty$ & nonlinear implementation
- **forming/following mode** enforced by small-signal Bode characterization
- **linear stability** under interconnection





[Huang, Xin, & Dörfler, '20],

[Chen, Zhou, Tayybei, Prieto, Dörfler, & Blaabjerg, '21]

Hybrid angle control = matching + oscillator control

$$\dot{\theta} = \omega^\star + c_1 \underbrace{\left(v_{\mathsf{dc}} - v_{\mathsf{dc}}^\star\right)}_{\text{matching control}} + c_2 \underbrace{\sin\left(\frac{\theta - \theta^\star}{2}\right)}_{{}^1/_2 \text{ oscillator control}}$$

[Tayyebi, Anta, & Dörfler '21], [Subotic & Groß '21]

Hybrid angle control = matching + oscillator control

$$\dot{\theta} = \omega^\star + c_1 \underbrace{\left(v_{\mathsf{dc}} - v_{\mathsf{dc}}^\star\right)}_{\text{matching control}} + c_2 \underbrace{\sin\left(\frac{\theta - \theta^\star}{2}\right)}_{{}^1/_2 \text{ oscillator control}}$$

$\rightarrow$ **matching + VOC**: robust + performance

- **almost global stability** for $c_2 > c_1$
- **compatibility**: local droop & stability with dc source or ac grid dynamics
- **tuning** c_1 & c_2 (robustness & performance)

[Tayyebi, Anta, & Dörfler '21], [Subotic & Groß '21]

Hybrid angle control = matching + oscillator control

$$\dot{\theta} = \omega^\star + c_1 \underbrace{\left(v_{\text{dc}} - v_{\text{dc}}^\star\right)}_{\text{matching control}} + c_2 \underbrace{\sin\left(\frac{\theta - \theta^\star}{2}\right)}_{{}^1/_2 \text{ oscillator control}}$$

$\rightarrow$ **matching + VOC**: robust + performance

- **almost global stability** for $c_2 > c_1$
- **compatibility**: local droop & stability with dc source or ac grid dynamics
- **tuning** c_1 & c_2 (robustness & performance)

implementation aspects

- needs voltage & current **measurements**
- active **current limitation** with guaranteed **stability** $\rightarrow$ check out Lyapunov proof ☺

[Tayyebi, Anta, & Dörfler '21], [Subotic & Groß '21]

Hybrid angle control = matching + oscillator control

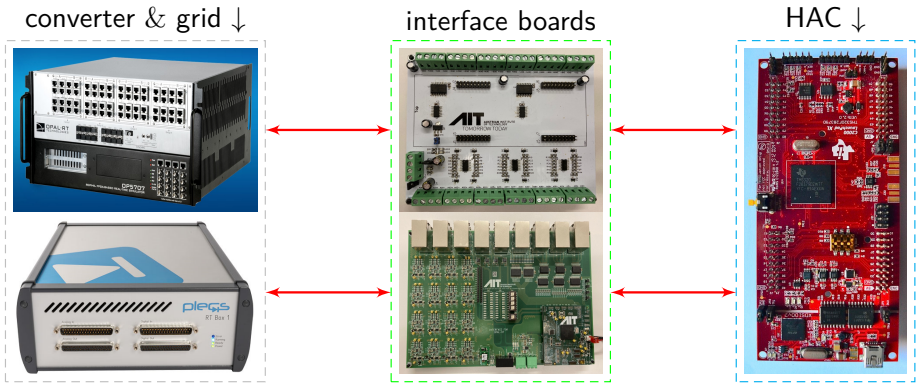
$$\dot{\theta} = \omega^\star + c_1 \underbrace{\left(v_{\text{dc}} - v_{\text{dc}}^\star\right)}_{\text{matching control}} + \underbrace{c_2 \sin\left(\frac{\theta - \theta^\star}{2}\right)}_{1/2 \text{ oscillator control}}$$

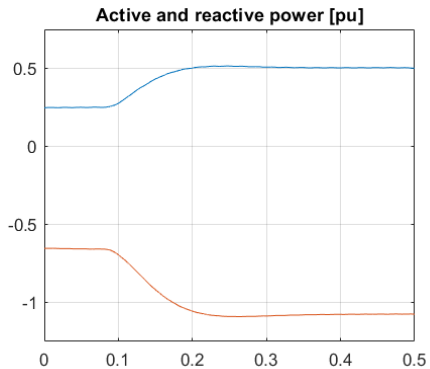
$\rightarrow$ **matching + VOC**: robust + performance

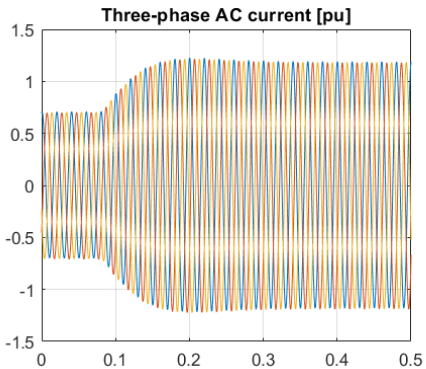
- **almost global stability** for $c_2 > c_1$
- **compatibility**: local droop & stability with dc source or ac grid dynamics
- **tuning** c_1 & c_2 (robustness & performance)

implementation aspects

- needs voltage & current **measurements**
- active **current limitation** with guaranteed **stability** $\rightarrow$ check out Lyapunov proof ☺

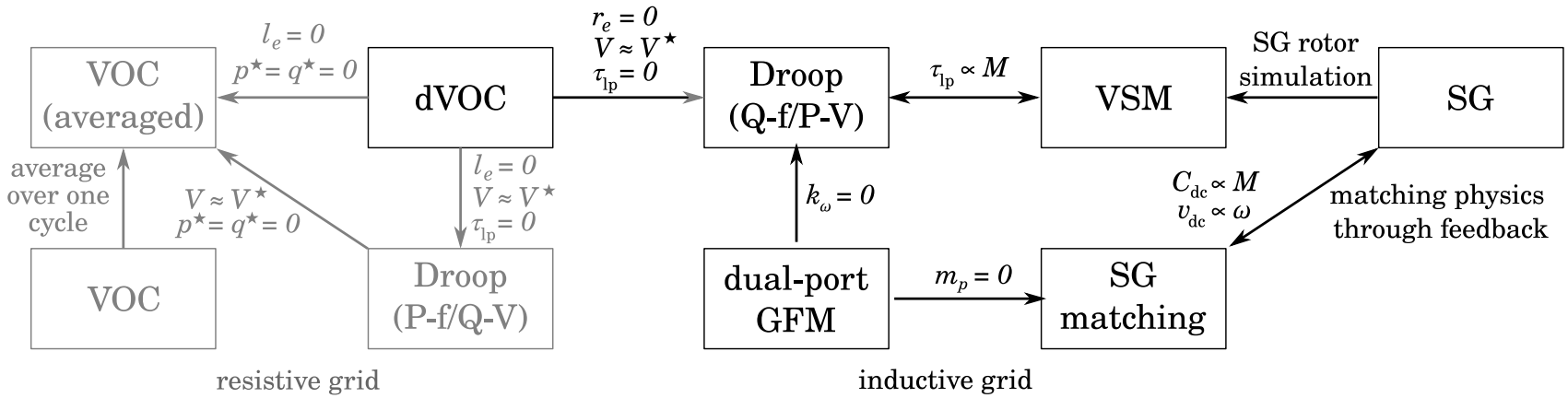






[Tayyebi, Anta, & Dörfler '21], [Subotic & Groß '21]

Often research goes in circles until we (hopefully) arrive at a bigger picture



SUMMARY OF CONTROL TRANSFER MATRICES CORRESPONDING TO DIFFERENT GRID-FORMING CONTROLLERS

Feedback Signals $\boldsymbol{y}$	v_{dc}	p	ω_u	q	V	v_{dc}	p	ω_u	q	V	v_{dc}	p	ω_u	q	V
Transfer Matrix ϕ_{ij}	ϕ_{11}	ϕ_{12}	ϕ_{13}	ϕ_{14}	ϕ_{15}	ϕ_{21}	ϕ_{22}	ϕ_{23}	ϕ_{24}	ϕ_{25}	ϕ_{31}	ϕ_{32}	ϕ_{33}	ϕ_{34}	ϕ_{35}
droop-1 [2], [3]	PI	0	0	0	0	0	P	0	0	0	0	0	0	P	0
droop-2 [8]	PI	0	0	0	0	0	0	0	P	0	0	I	0	0	I
droop-3 [3], [30]	PI	0	0	0	0	0	IF	0	0	0	0	0	0	0	0
droop-4 [11]	PI	0	0	0	0	0	PD	0	0	0	0	0	0	P	0
droop-5 [4]	PI	0	0	0	0	0	P{IF×D}	0	0	0	0	0	0	P	0
PSC-1 [3]	PI	0	0	0	0	0	P	0	0	0	0	0	0	P	0
PSC-2 [12]	PI	0	0	0	0	0	IF×PD	0	0	0	0	0	0	I	0
PSC-3 [13]	PI	0	0	0	0	0	IF×PD	0	0	0	0	0	0	I	0
VSG-1 [22], [31]	PI	0	0	0	0	0	IF	0	0	0	0	0	0	0	IF
VSG-2 [21], [22], [29]	PI	0	0	0	0	0	IF	0	0	0	0	0	0	PI	PI
VSG-3 [17]	PI	0	0	0	0	0	IF	0	0	0	0	0	P	P	0
VSG-4 [32]	PI	0	0	0	0	0	IF	IF	0	0	0	0	0	P	0
VSG-5 [4], [21], [33]	PI	0	0	0	0	0	IF	IF	0	0	0	0	0	PI	PI
VSG-6 [16]	PI	0	0	0	0	0	IF	0	0	IF	0	0	0	I	I
VSG-7 [15]	PI	0	0	0	0	0	O×PD	0	0	0	0	0	0	I	I
VSG-8 [4]	PI	0	0	0	0	0	IF×PD	0	0	0	0	0	0	PI	PI
VSG-9 [19]	PI	0	0	0	0	IF×PD	IF×PD	0	0	0	0	0	0	P	0
VSG-10 [21]	PI	0	0	0	0	0	IF_1{IF_1×IF_2×D}	0	0	0	0	0	0	PI	PI
VSG-11 [23]	PI	0	0	0	0	0	O×PD{O×IF×PD×D}	0	0	0	0	0	0	I	I
VSG-12 [20], [21]	PI	0	0	0	0	0	O×PD_1{O×IF×PD_2×D}	0	0	0	0	0	0	PI	PI
matching-1 [5]	P	0	0	0	0	P	0	0	0	0	0	0	0	0	PI
matching-2 [18]	0	0	0	0	0	P	0	0	0	0	P	0	0	0	0
Generated Inputs $\boldsymbol{u}$	i_u					ω_u					E_u				

Control of Low-Inertia Power Systems

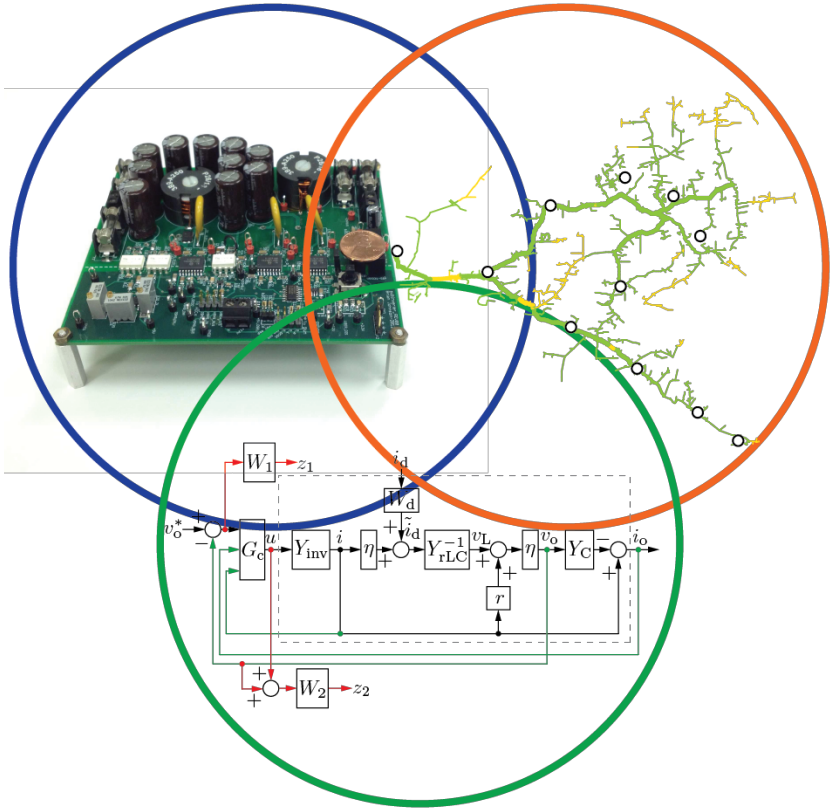
Florian Dörfler[1] and Dominic Groß[2]

[1]Automatic Control Laboratory, ETH Zurich, Zurich, Switzerland, 8092; email: dorfler@ethz.ch

[2]Electrical and Computer Engineering, University of Wisconsin-Madison, Madison, United States, WI 53706; email: dominic.gross@wisc.edu

Exciting research bridging communities

power electronics

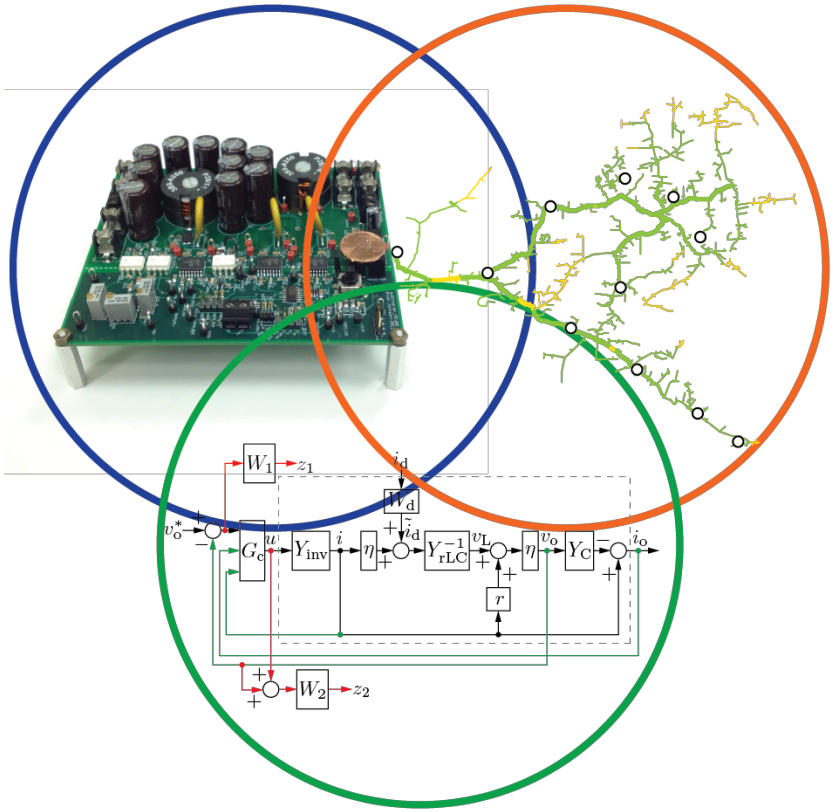


power systems

control systems

Exciting research bridging communities

power electronics



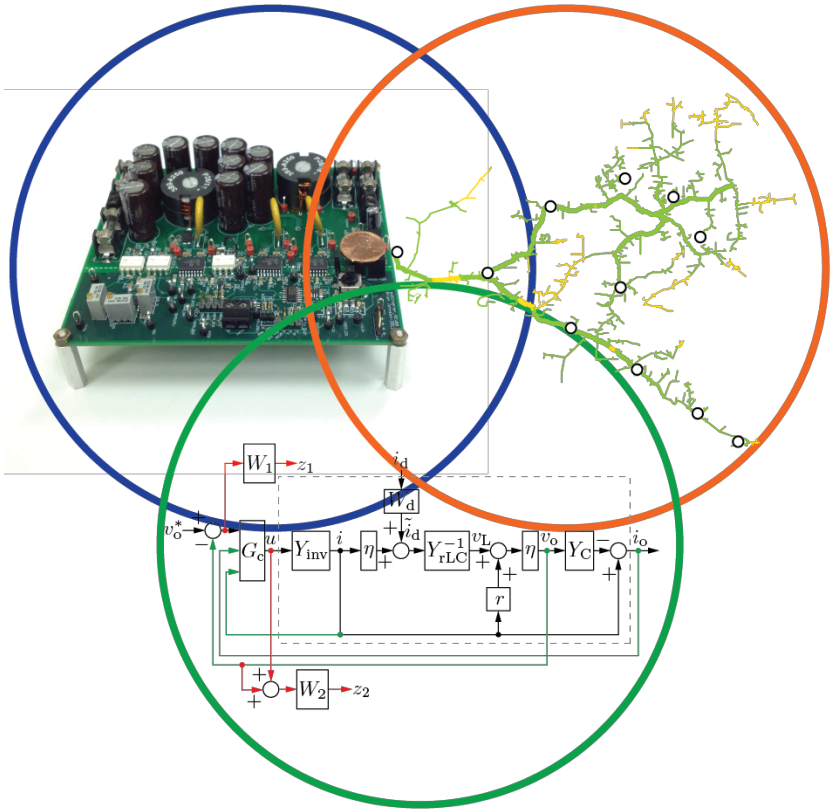
power systems

control systems

take-home messages: ① converter $\neq$ flywheel, ② multivariable design instead of decoupling, ③ take current constraints & ④ dc voltage into account

Exciting research bridging communities

power electronics



power systems

control systems

take-home messages: ① converter $\neq$ flywheel, ② multivariable design instead of decoupling, ③ take current constraints & ④ dc voltage into account . . . & ⑤ sync is only the beginning: what to do once sync'd ?

finally . . . recall


POWER IS NOTHING WITHOUT CONTROL